



# Topology Optimization for Computational Fabrication

Jun Wu, Niels Aage, Sylvain Lefebvre, Charlie Wang

<https://topopt.weblog.tudelft.nl/>



# Topology Optimization for Computational Fabrication

## Part 3: Topology Optimization with Controllable Geometric Features

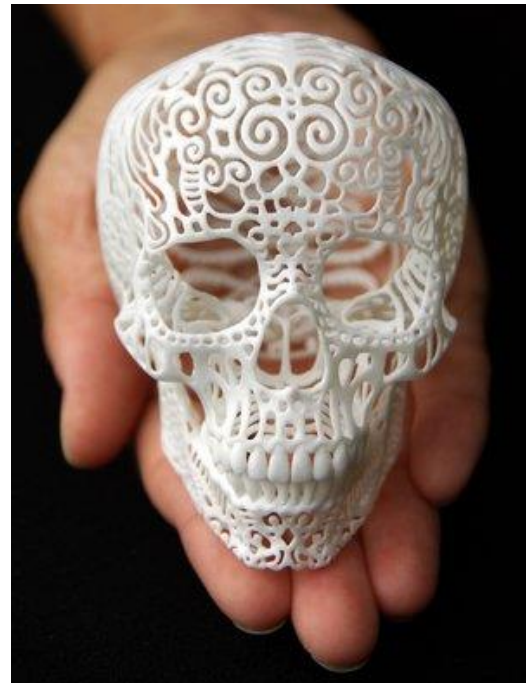
Dr. Jun Wu

TU Delft

# Additive Manufacturing: Complexity is free



TU Delft & MX3D, 2015



Joshua Harker

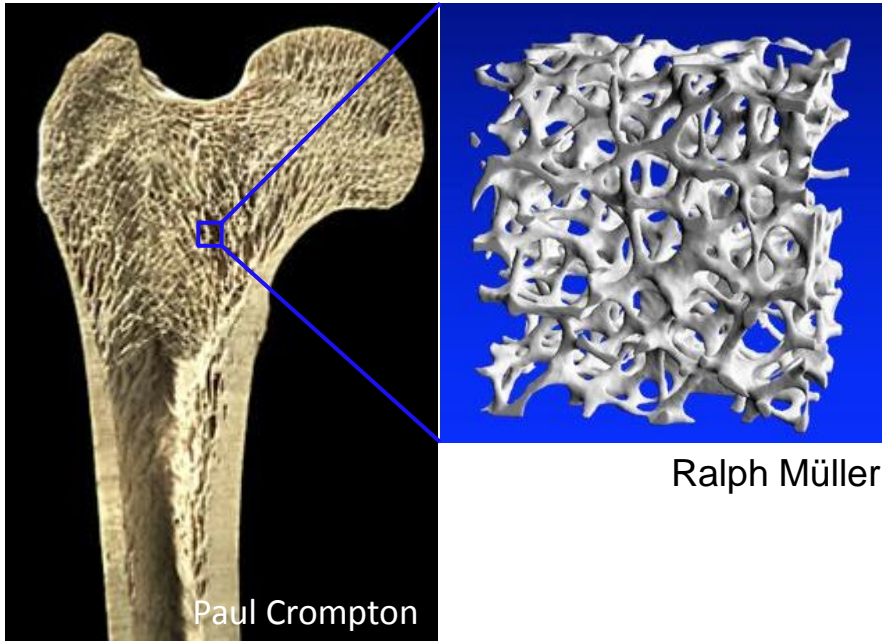


Scott Summit

# Complexity is free? ... Not really!

- Printer resolution: Minimum geometric feature size
- Layer-upon-layer: Supports for overhang region
- Shell-infill composite

Tiny details



Supports



Concept Laser GmH

Infill

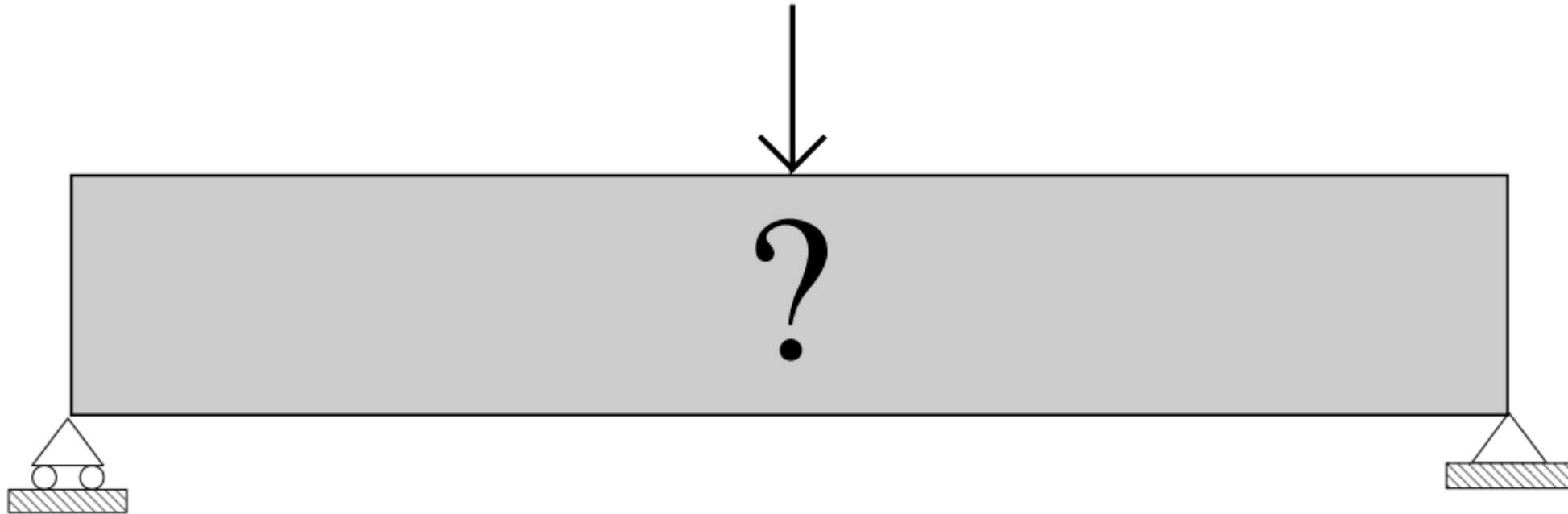


[mpi.fs.tum.de](http://mpi.fs.tum.de)

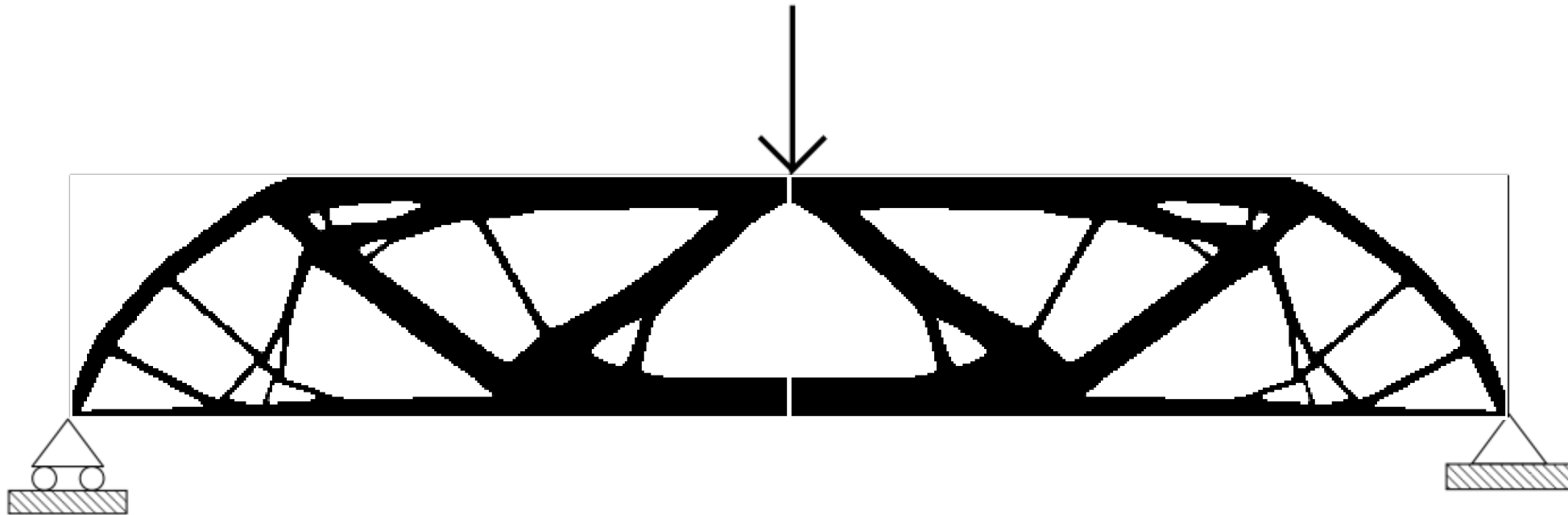
# Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**

Test case



## Test case



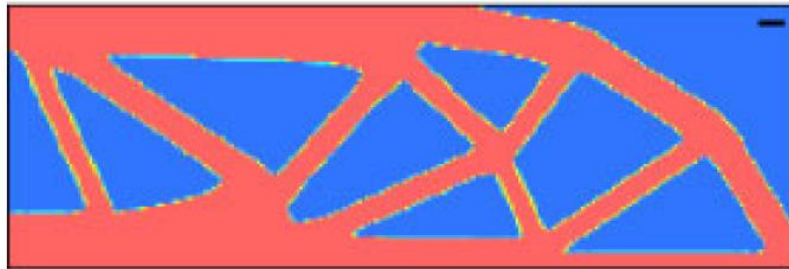
# Geometric feature control by density filters

(An incomplete list)

Reference



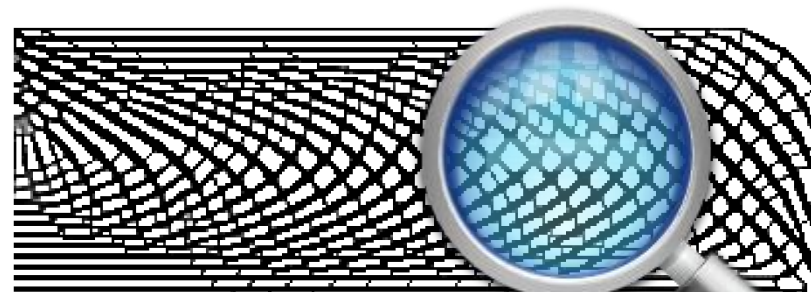
Minimum feature size, Guest'04



Coating structure, Clausen'15



Self-supporting design, Langelaar'16



Porous infill, Wu'16

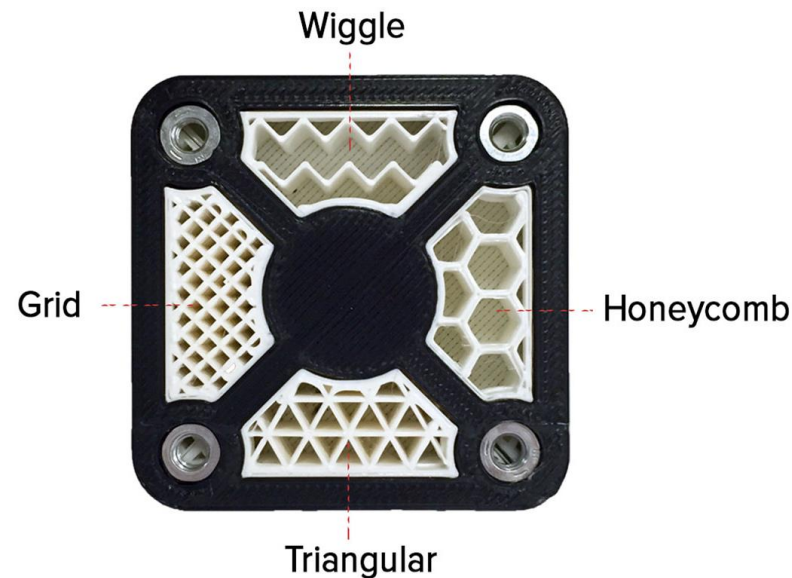


# Infill in 3D Printing

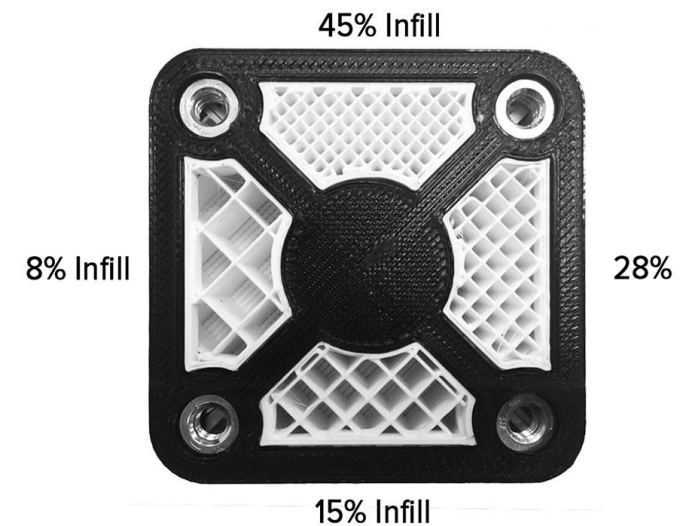
- A user-selected **regular** pattern, with a volume percentage
- A rough balance between
  - Physical properties (mass, strength), and
  - Cost (material usage, print time)



Infill



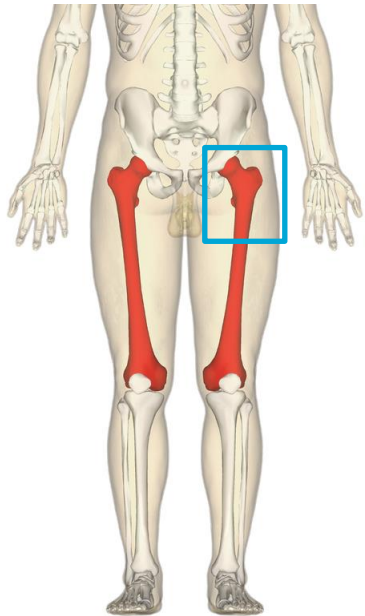
Different infill patterns



Different infill percentages

# Infill in Nature

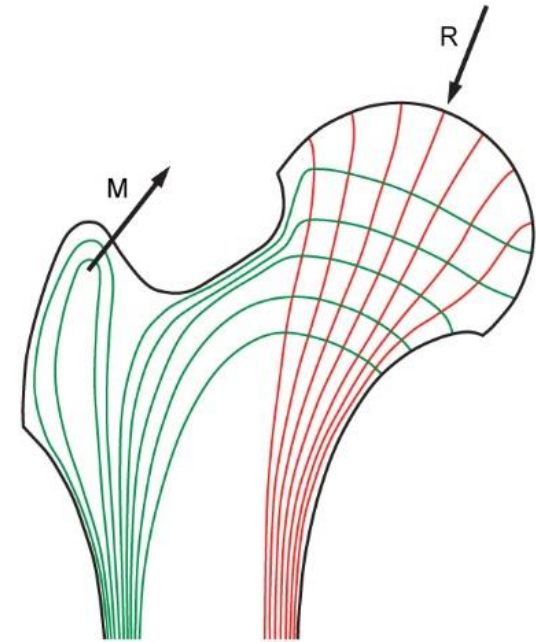
- Trabecular bone
  - Porous structures, oriented with the principle stress direction
  - Resulted from a natural optimization process
  - Light-weight-high-resistant



wikipedia.org



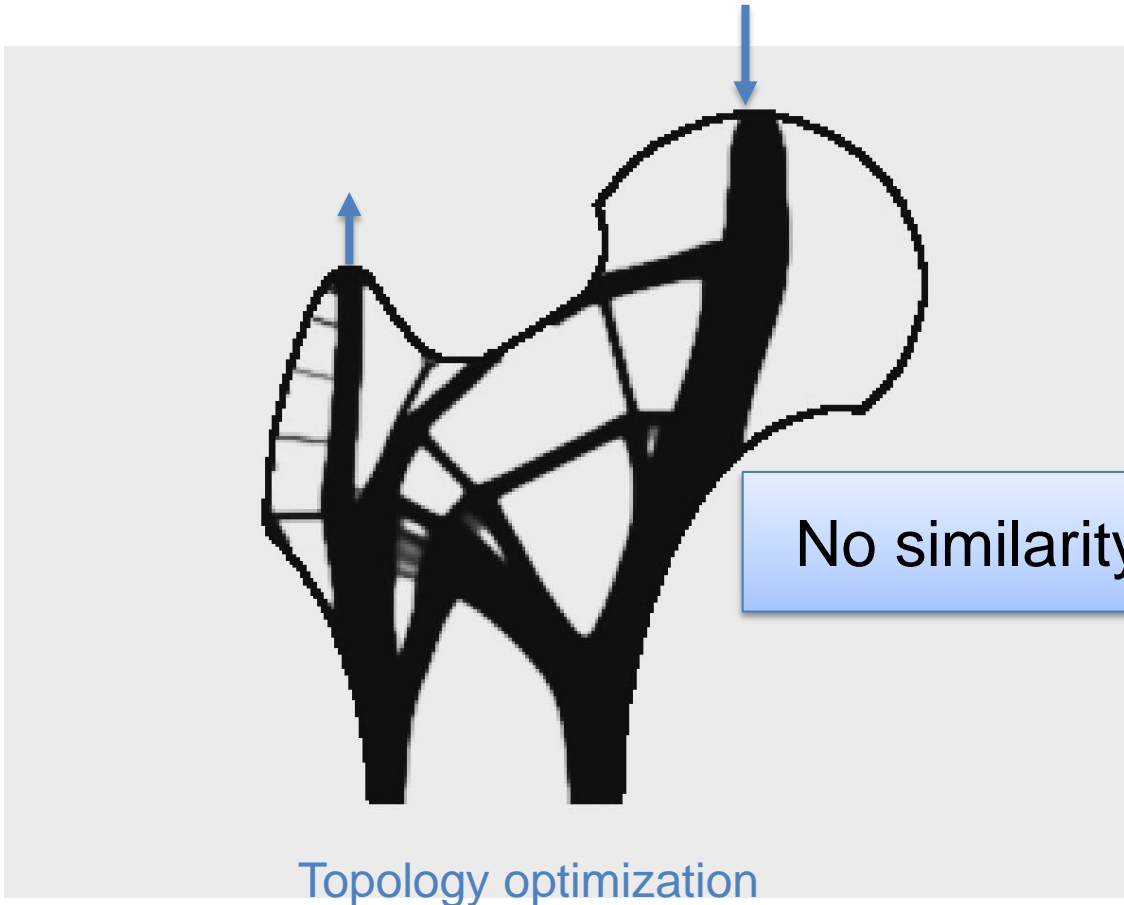
Cross-section of  
a human femur



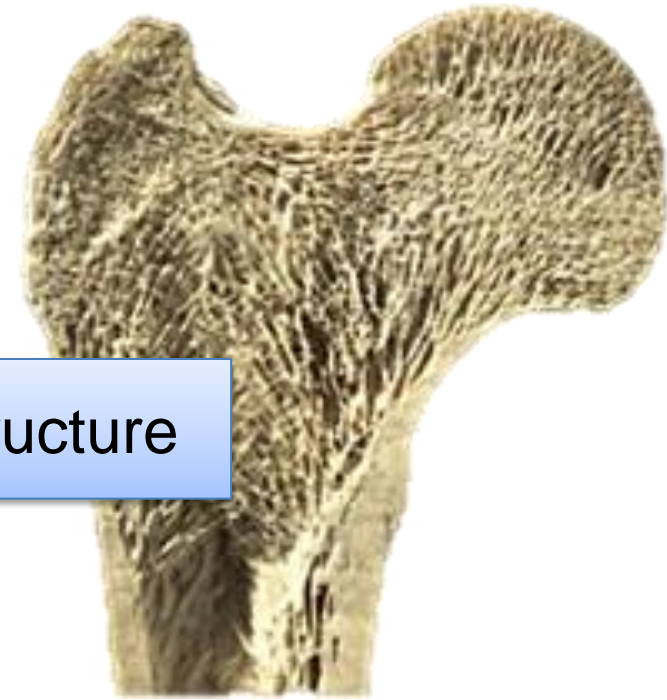
Principle stress directions

Optimize bone-like structures as infill for AM?

# Topology Optimization Applied to Design Infill



No similarity in structure



Infill in the bone

# Topology Optimization Applied to Design Infill

- Materials accumulate to “important” regions
- The **total** volume  $\sum_i \rho_i v_i \leq V_0$  does not restrict local material distribution



Infill by standard  
topology optimization



Infill in the bone

# Approaching Bone-like Structures: The Idea

- Impose **local constraints** to avoid fully solid regions

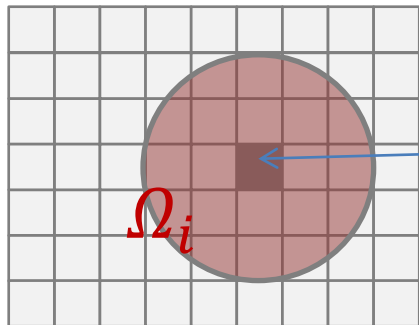
$$\text{Min:} \quad c = \frac{1}{2} U^T K U$$

$$\text{s.t. :} \quad K U = F$$

$$\rho_i \in [0,1], \forall i$$

~~$$\sum_i \rho_i \leq V_0$$~~

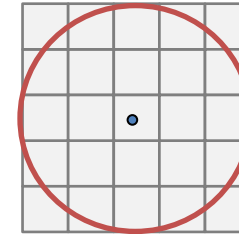
$$\hat{\rho}_i \leq \alpha, \forall i$$



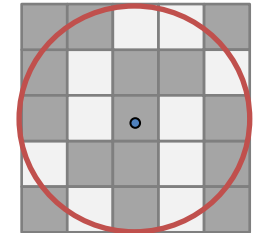
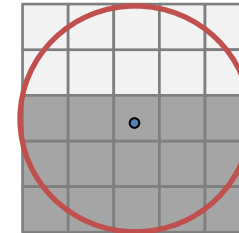
$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

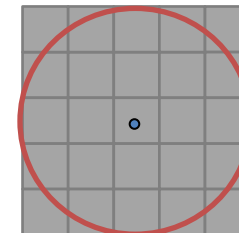
$$\hat{\rho}_i = 0.0$$



$$\hat{\rho}_i = 0.6$$



$$\hat{\rho}_i = 1.0$$



# Constraints Aggregation (Reduce the Number of Constraints)

$$\hat{\rho}_i \leq \alpha, \forall i$$



$$\max_{i=1,\dots,n} |\hat{\rho}_i| \leq \alpha$$



$$\lim_{p \rightarrow \infty} \|\rho\|_p = (\sum_i (\hat{\rho}_i)^p)^{\frac{1}{p}} \leq \alpha$$

Too many constraints!

A single constraint  
But non-differentiable

A single constraint  
and differentiable  
Approximated with  $p = 16$

# Optimization Process: The same as in the standard toptopt

- Impose **local constraints** to avoid fully solid regions

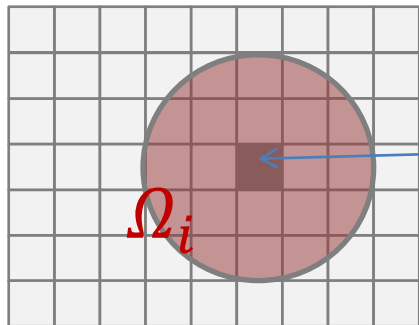
$$\text{Min: } c = \frac{1}{2} U^T K U$$

$$\text{s.t. : } K U = F$$

$$\rho_i \in [0,1], \forall i$$

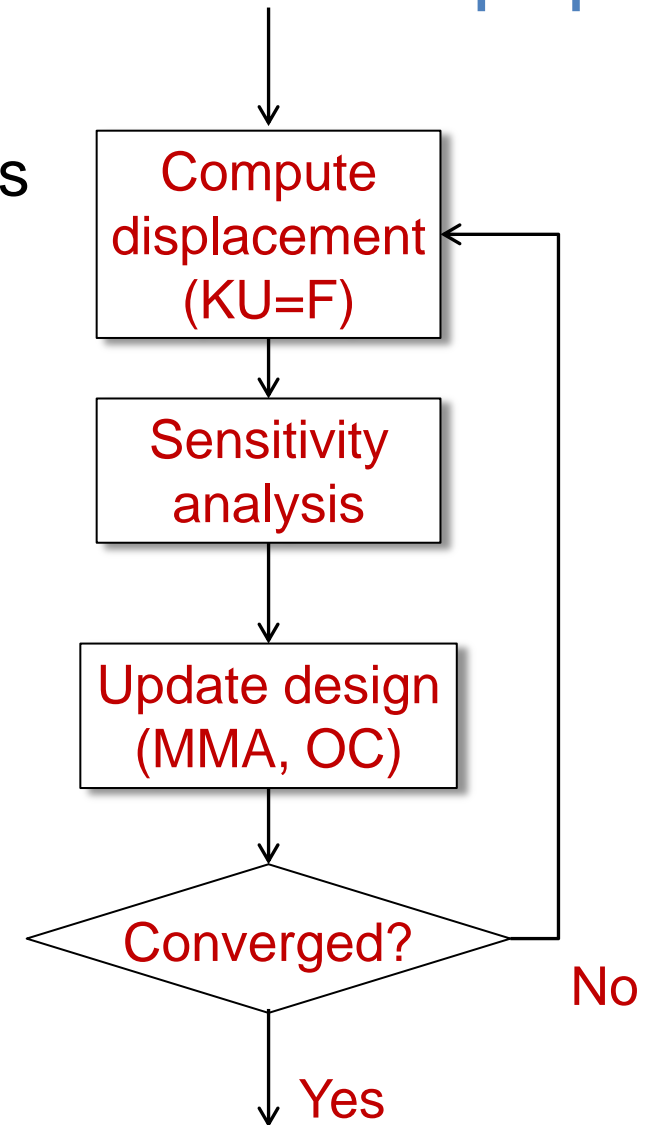
~~$$\sum_i \rho_i \leq V_0$$~~

$$\hat{\rho}_i \leq \alpha, \forall i$$



$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

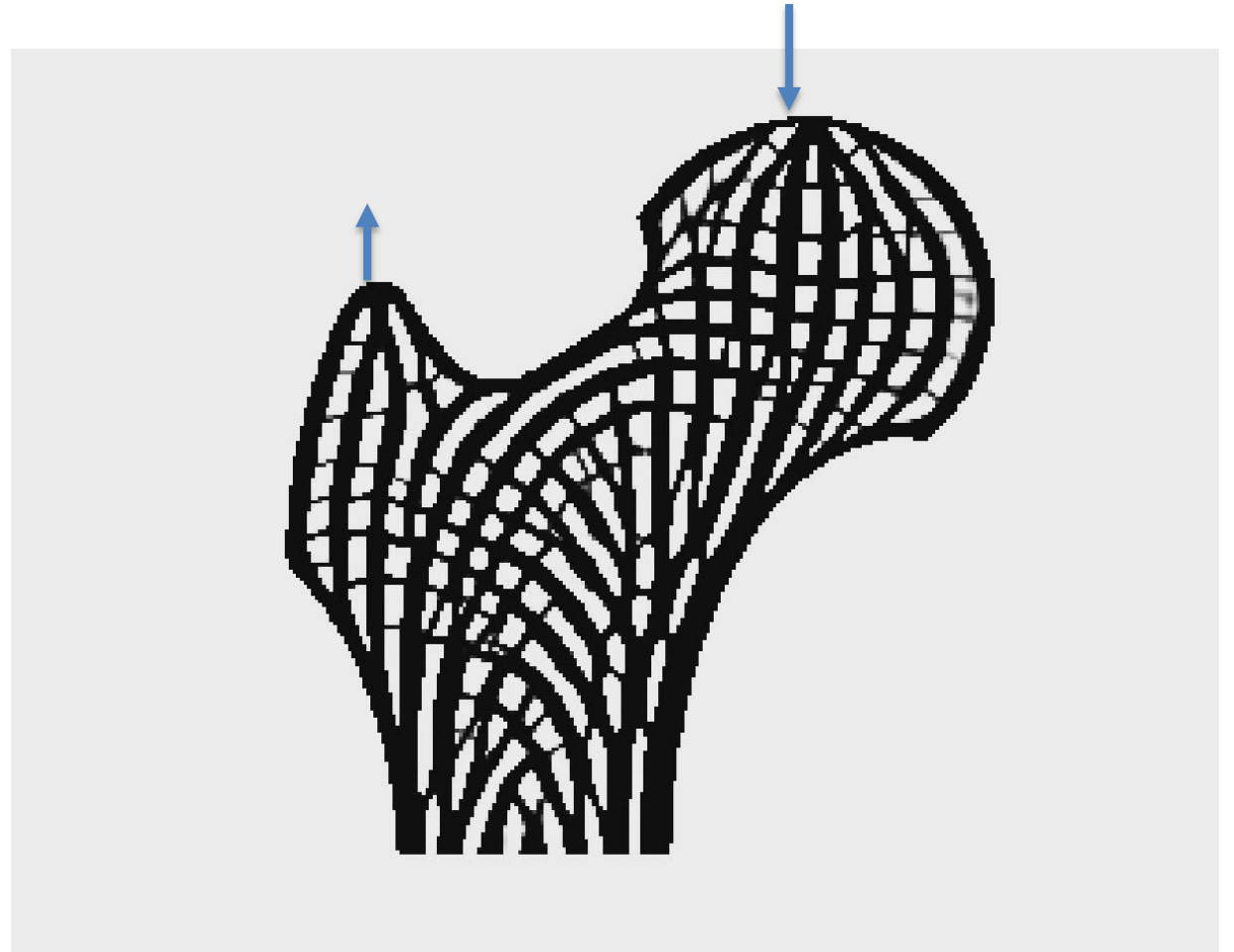




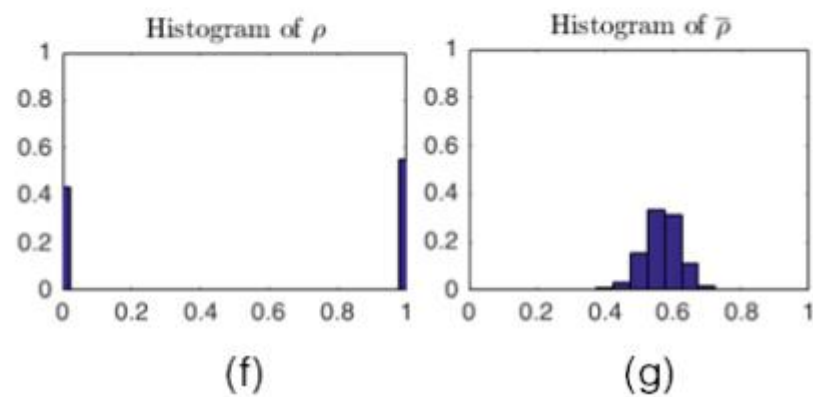
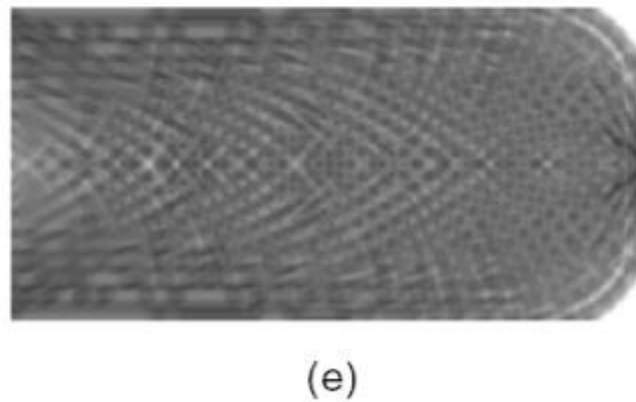
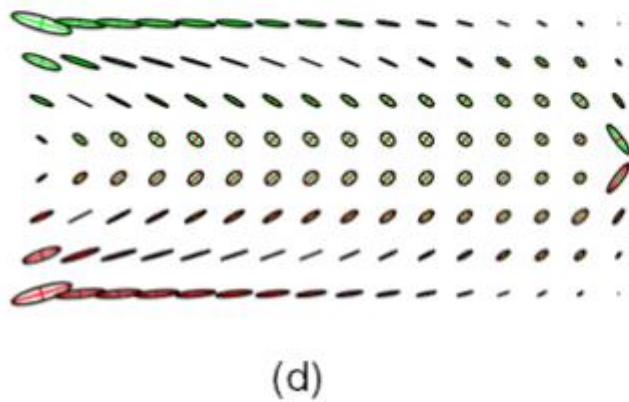
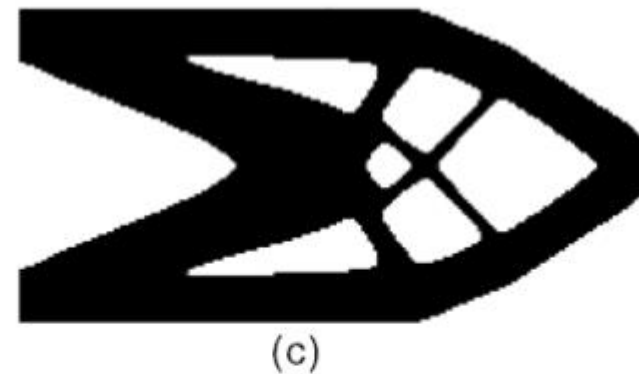
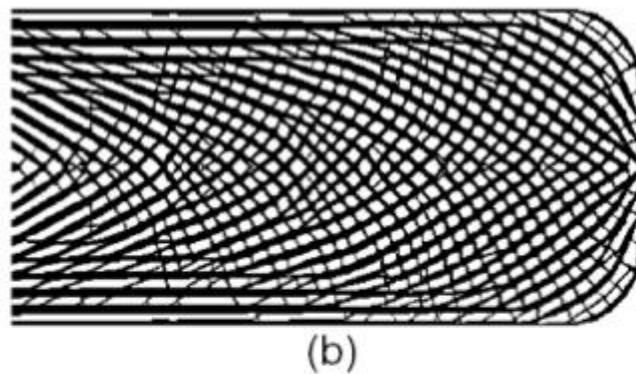
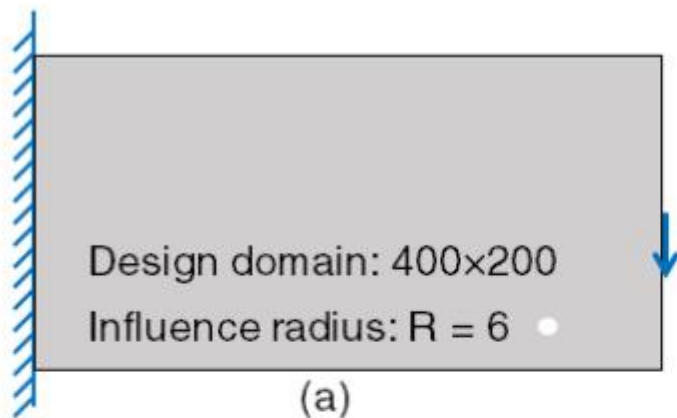
## Bone-like Infill in 2D



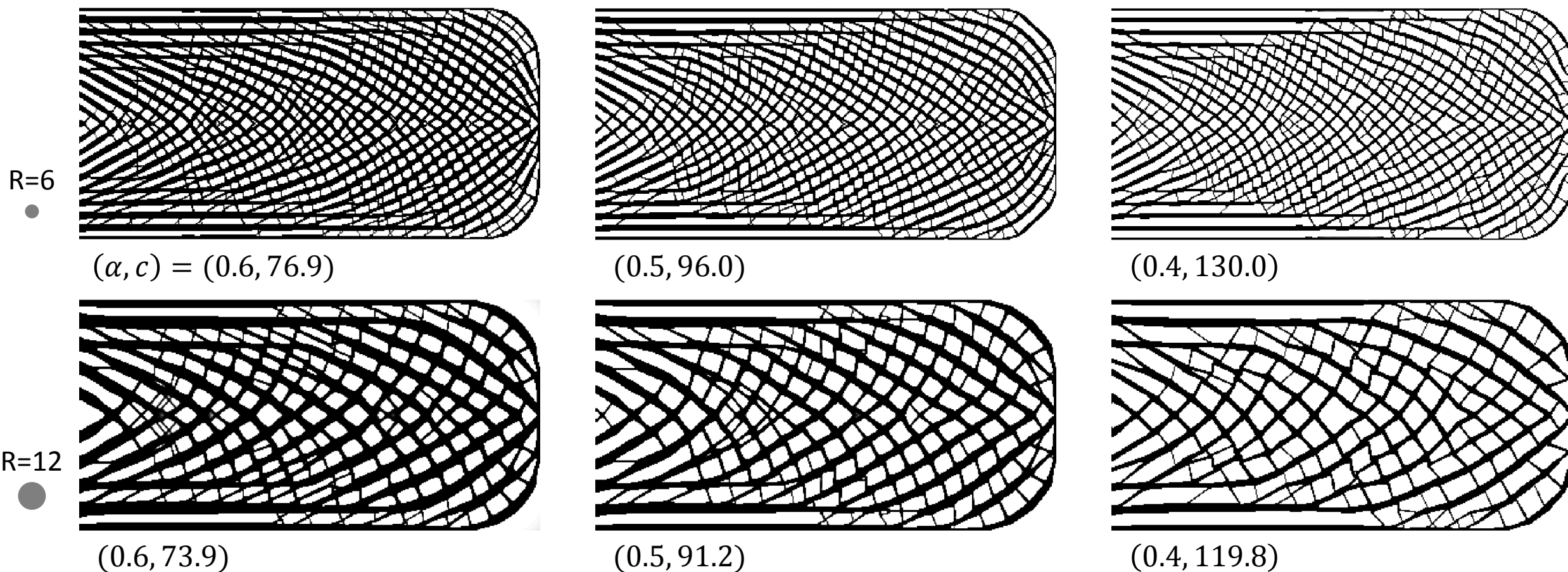
Cross-section of a human femur



# A Test Example

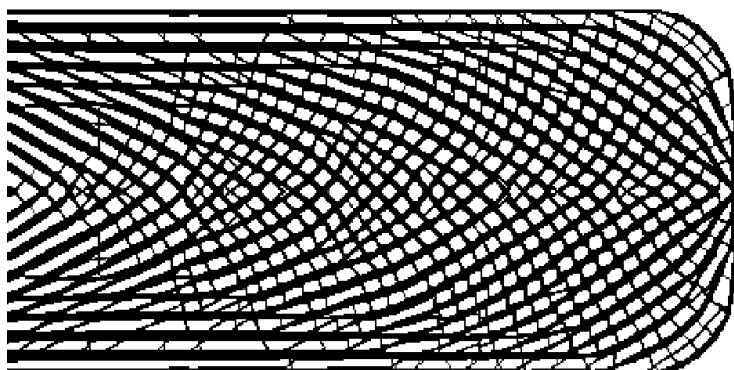


# Effects of Filter Radius and Local Volume Upper Bound

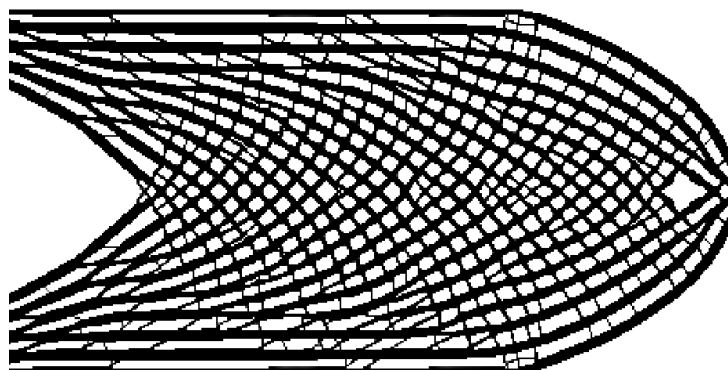


# Local and Global Volume Constraints

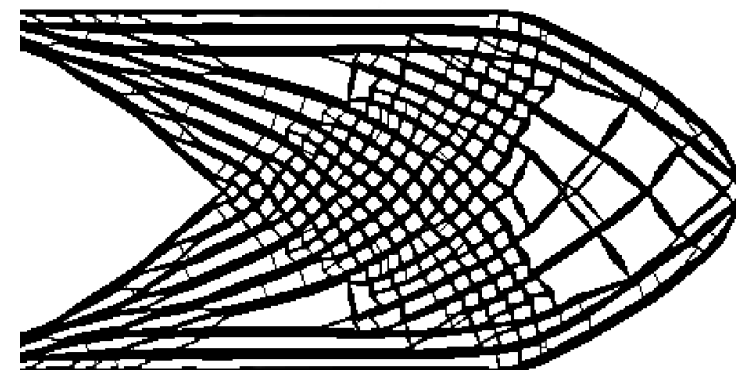
R=6



$(\alpha, \alpha_{total}, c) = (0.6, 0.56, 76.9)$



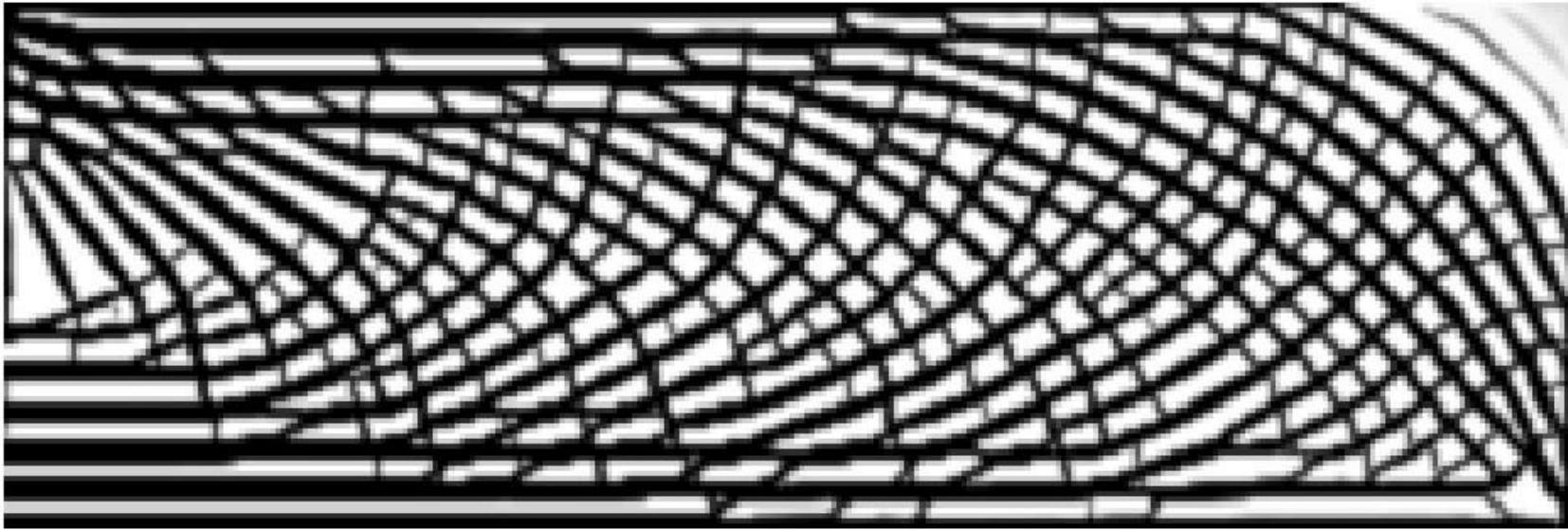
$(0.6, 0.50, 79.1)$



$(0.6, 0.40, 94.0)$

## Result: 2D Animation

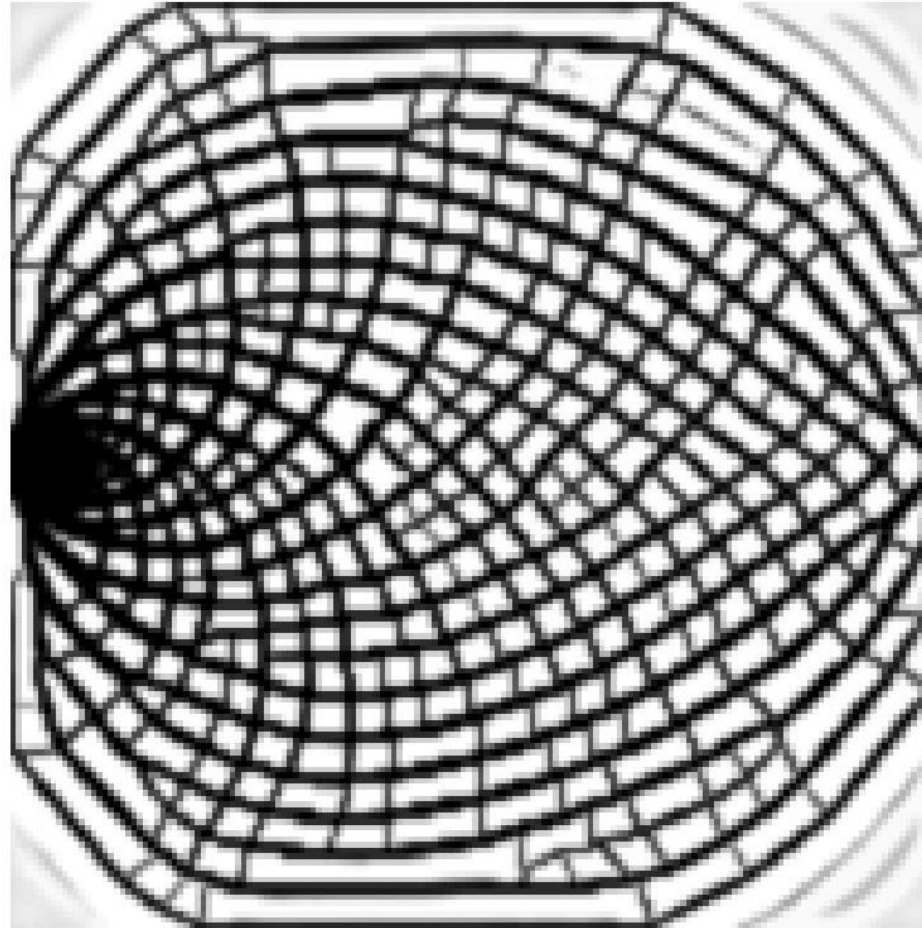
xPhys





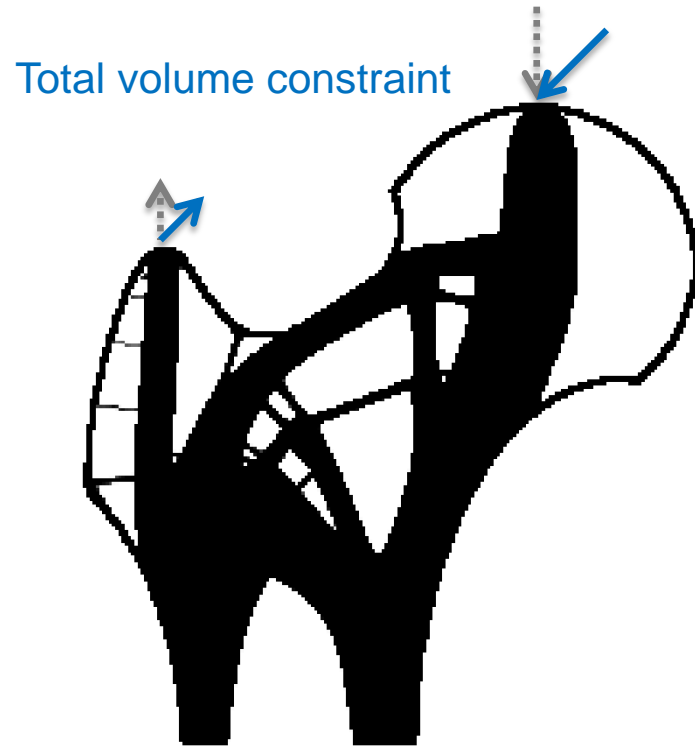
## Result: 2D Animation

xPhys

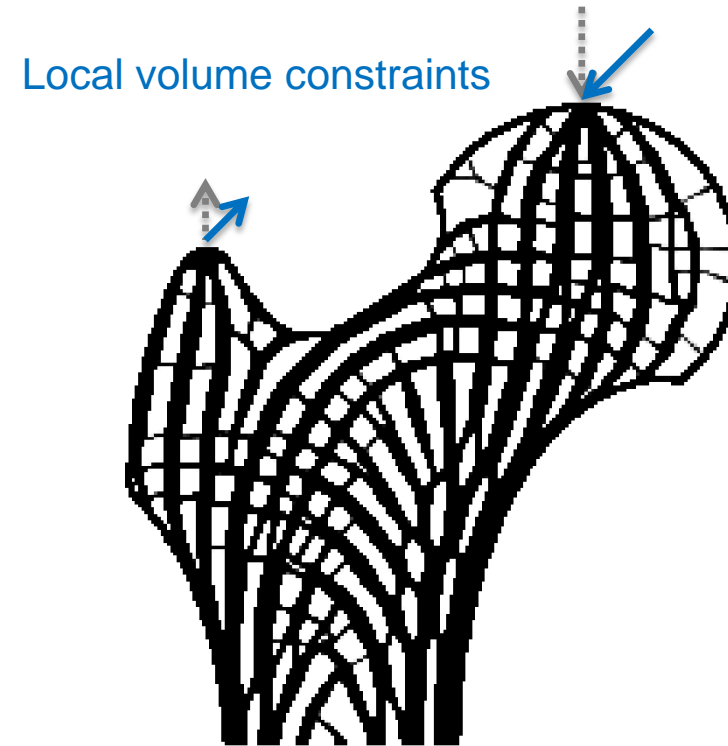


# Robustness wrt. Force Variations

- Bone-like structures are significantly stiffer (126%) in case of **force variations**



$c = 30.54$   
 $c' = 45.83$

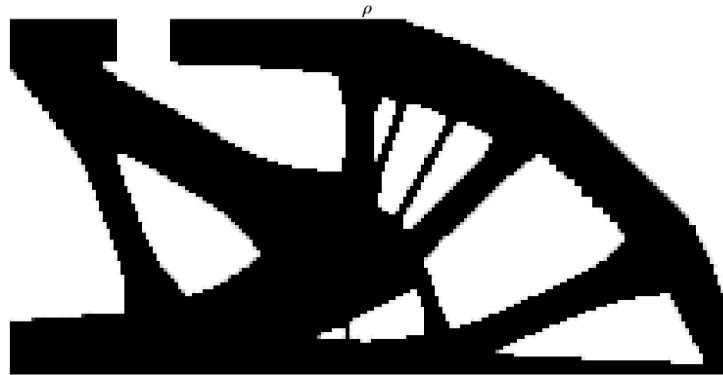


$c = 36.72$   
 $c' = 36.23$

# Robustness wrt. Material Deficiency

- Bone-like structures are significantly stiffer (180%) in case of [material deficiency](#)

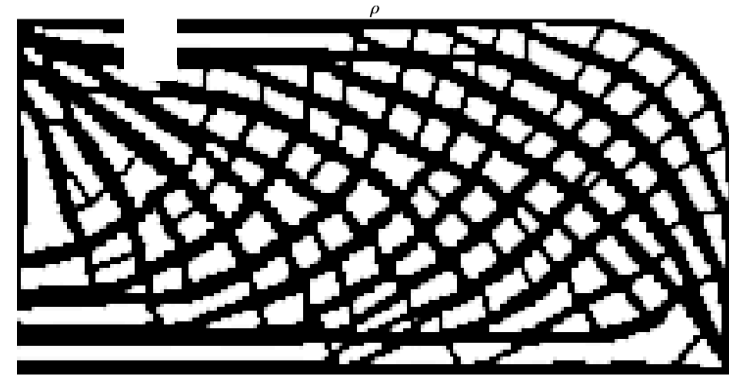
Total volume constraint



$$c = 76.83$$

$$c' = 242.77$$

Local volume constraints



$$c = 93.48$$

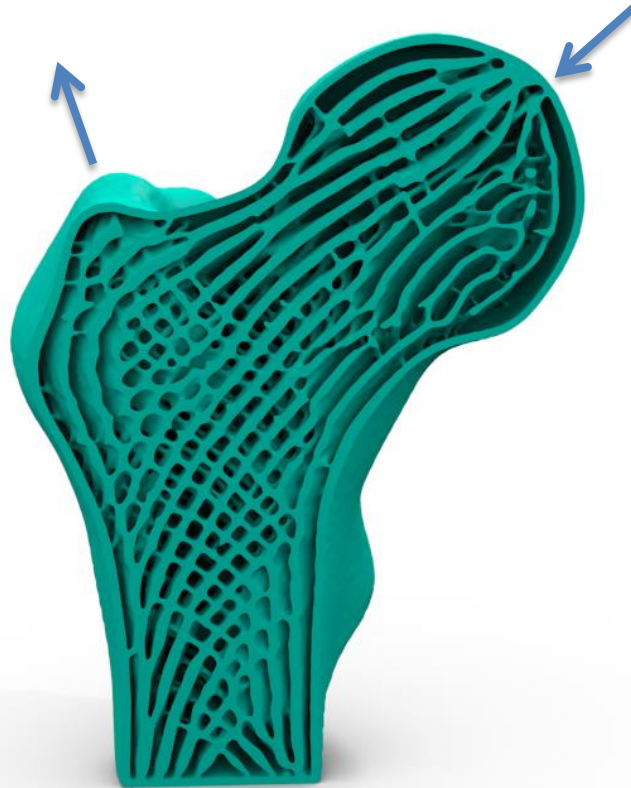
$$c' = 134.84$$



## Bone-like Infill in 3D



Infill in the bone



Optimized bone-like infill



# FDM Prints



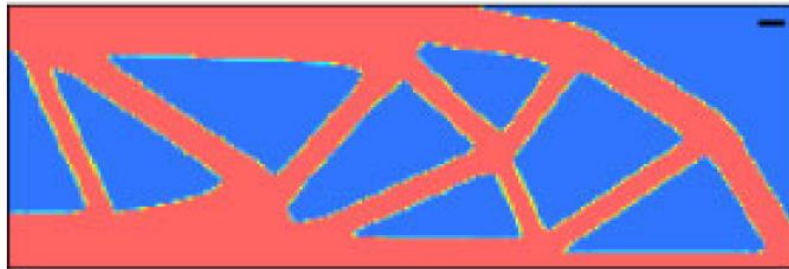
# Geometric feature control by density filters

(An incomplete list)

Reference



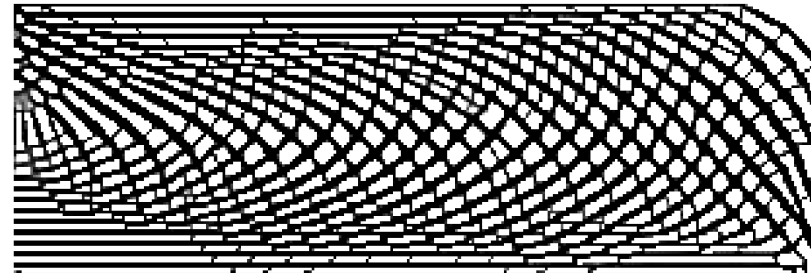
Minimum feature size, Guest'04



Coating structure, Clausen'15



Self-supporting design, Langelaar'16



Porous infill, Wu'16

# Concurrent Shell-Infill Optimization

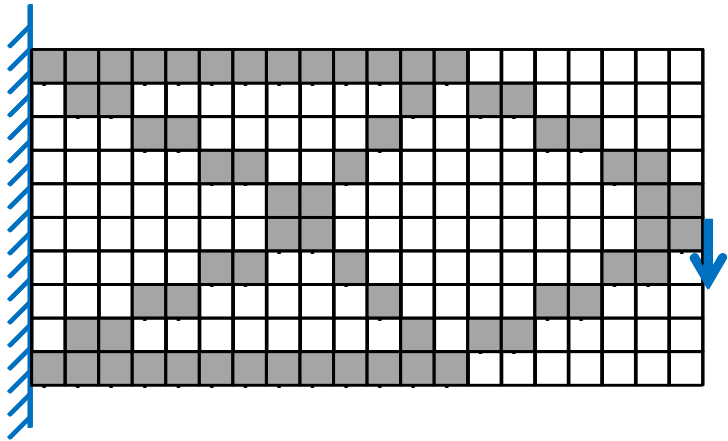


# Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**



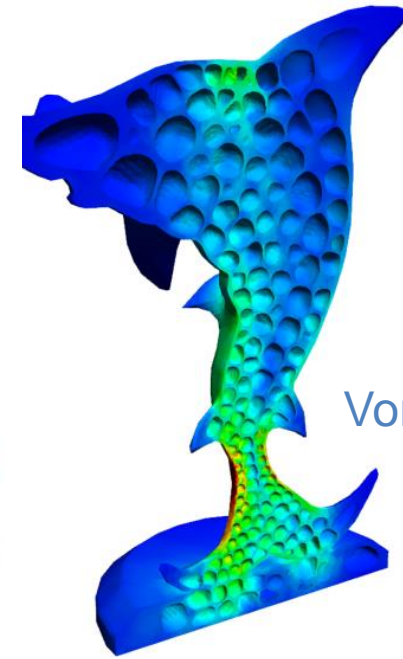
# Geometric feature control by alternative parameterizations (An incomplete list)



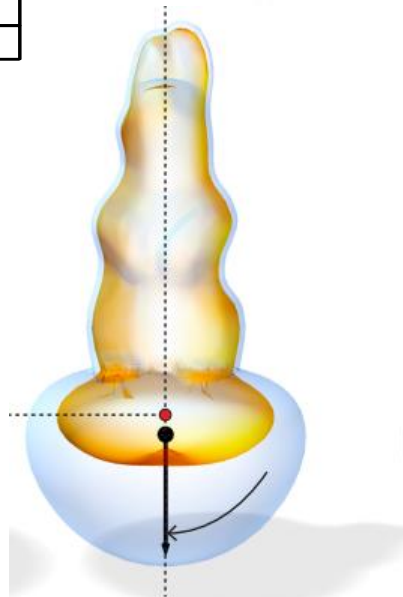
Reference: Voxel discretization



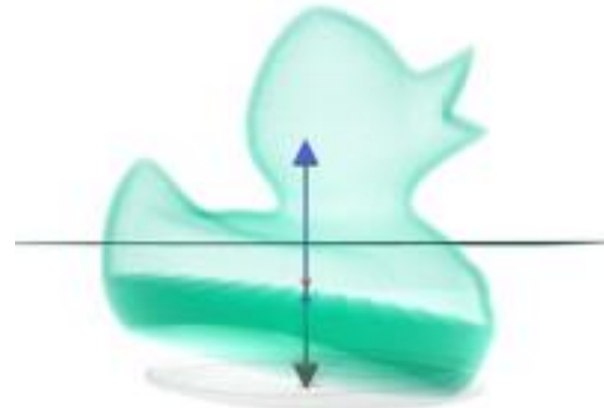
Skin-frame, Wang'13



Voronoi cells, Lu'14



Offset surfaces, Musialski'15



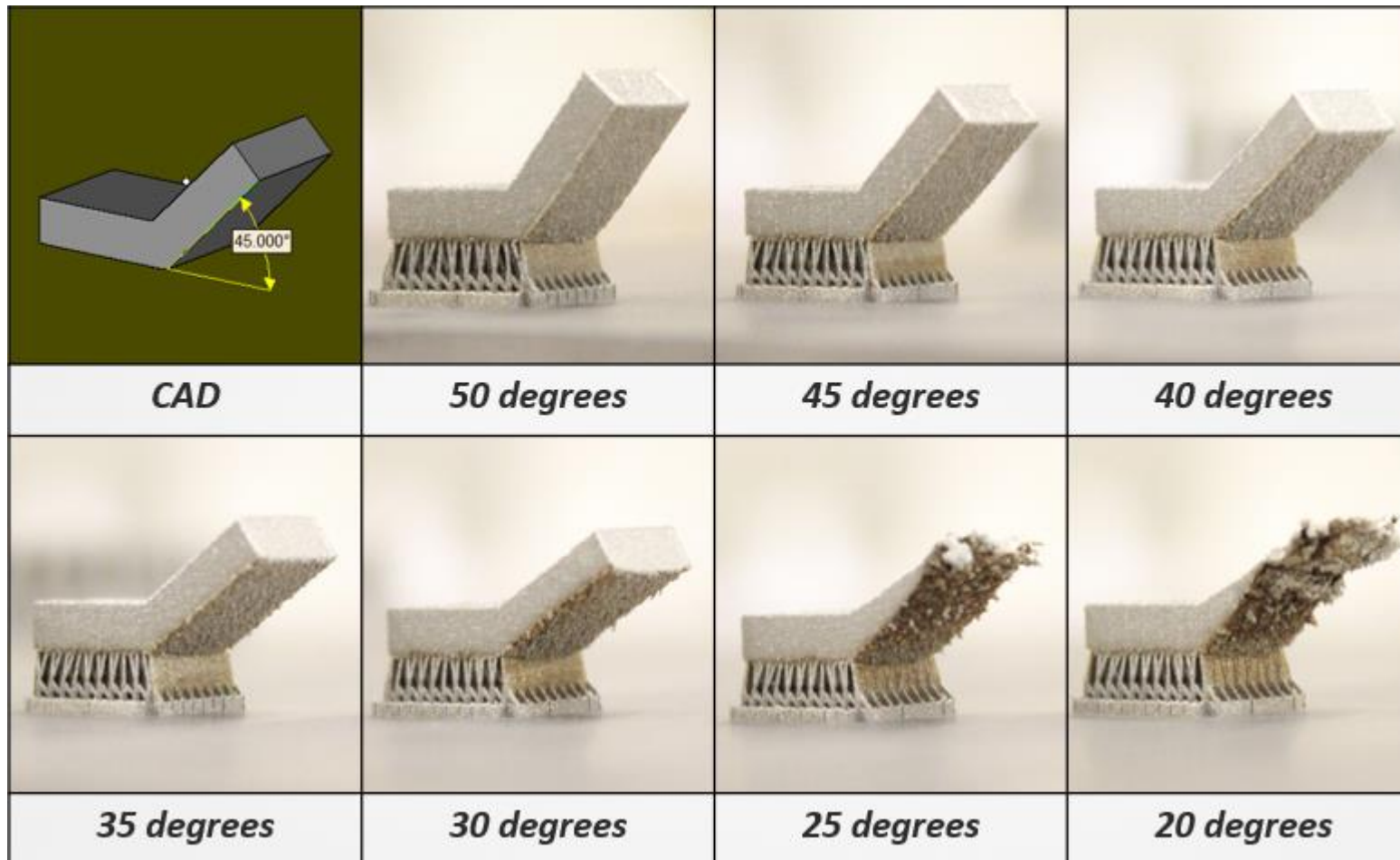
Ray representation, Wu'16



Adaptive rhombic, Wu'16

# Overhang in Additive Manufacturing

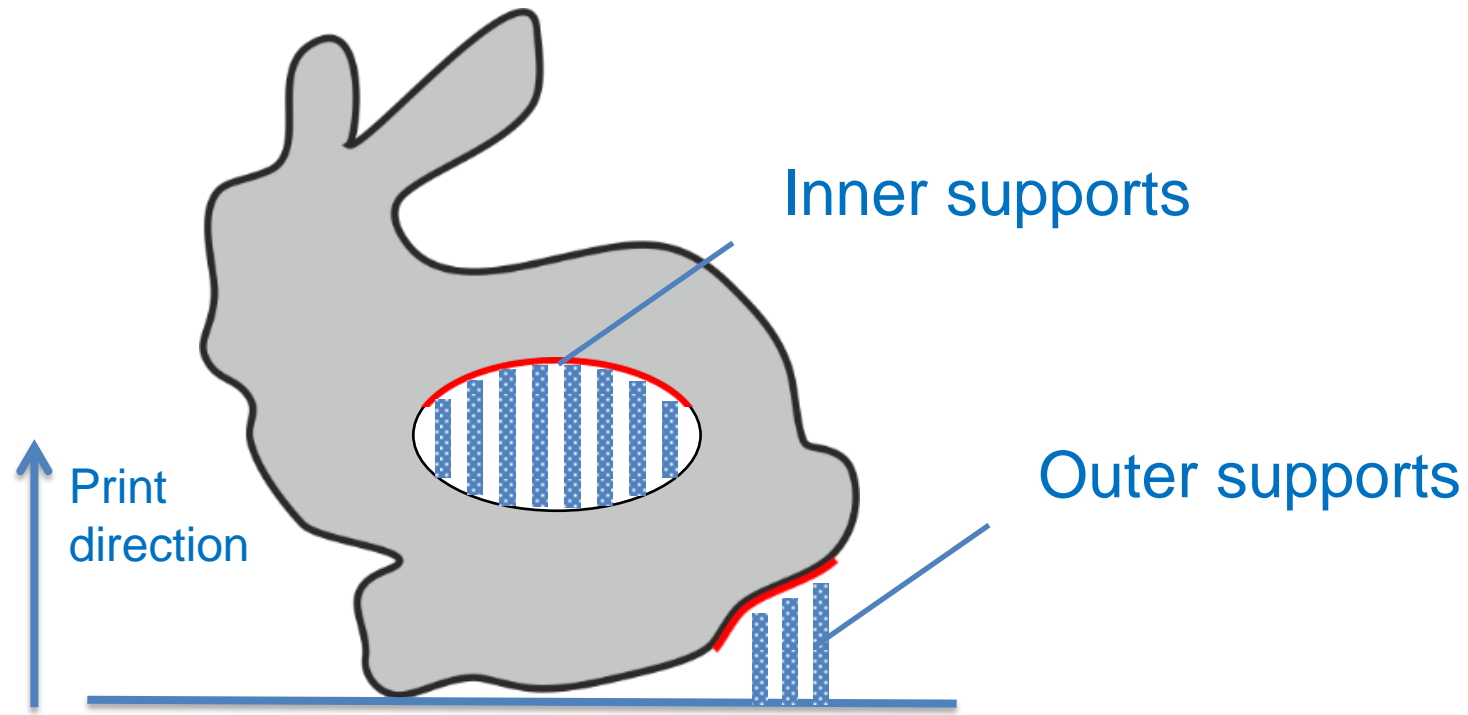
- Support structures are needed beneath overhang surfaces



<https://www.protolabs.com/blog/tag/direct-metal-laser-sintering/>

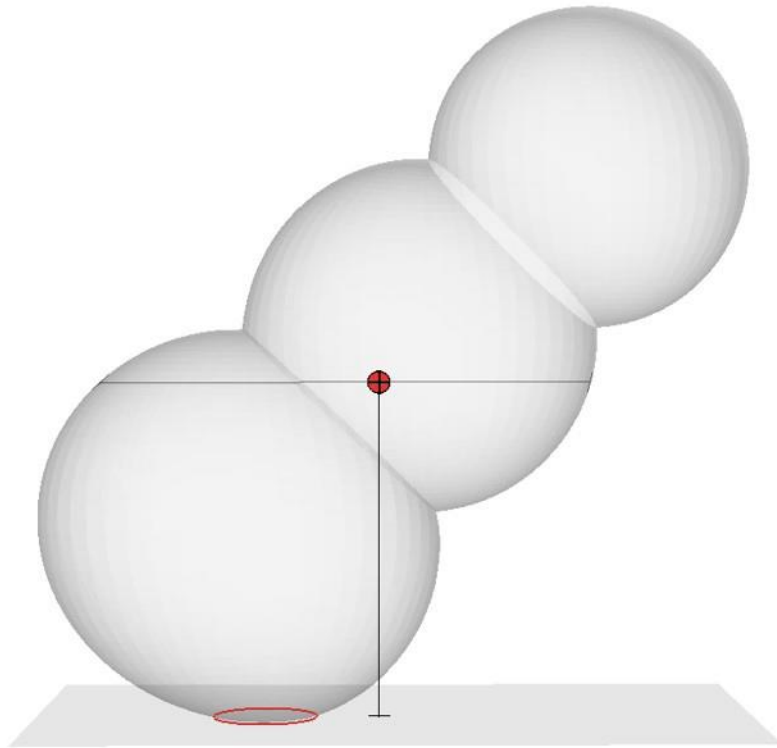
# Support Structures in Cavities

- Post-processing of **inner** supports is problematic

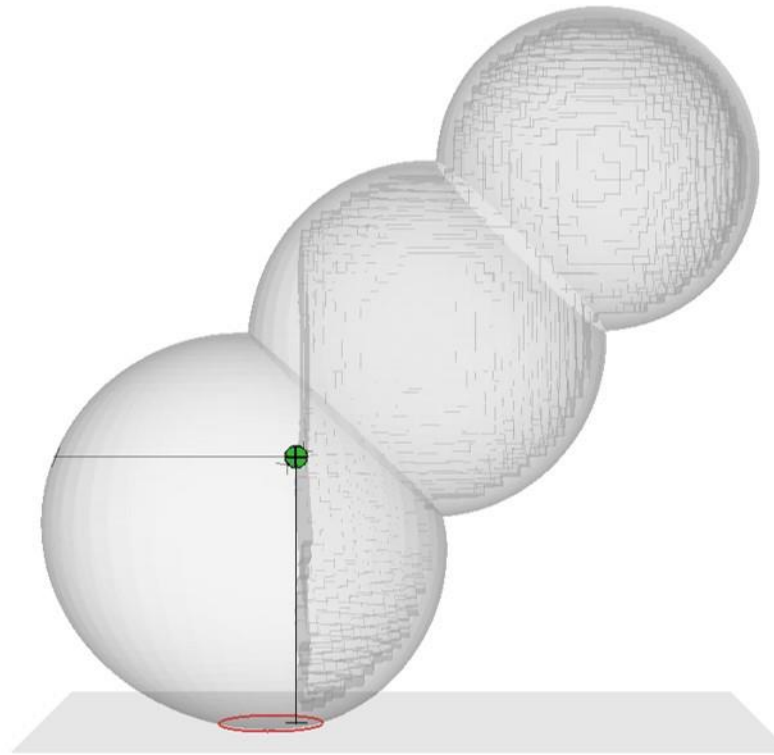




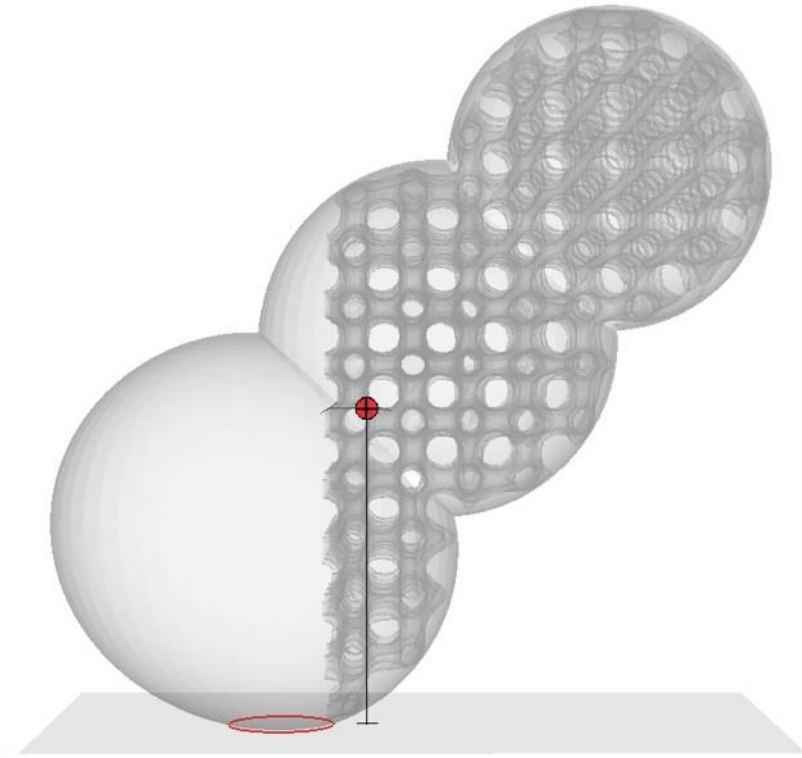
# Infill & Optimization Shall Integrate



Solid,  
Unbalanced



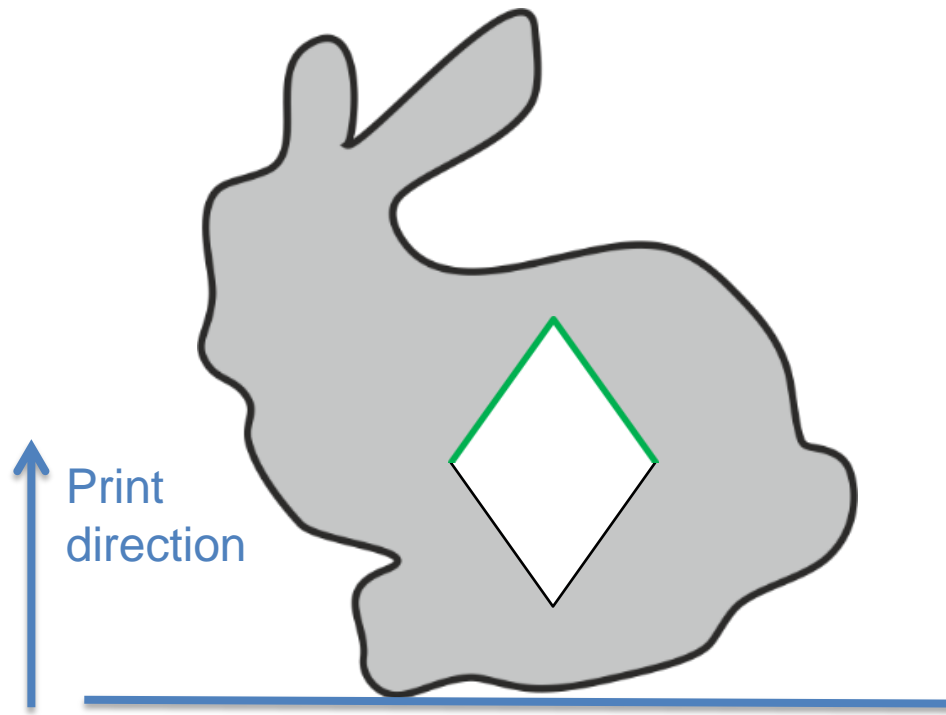
Optimized,  
Balanced



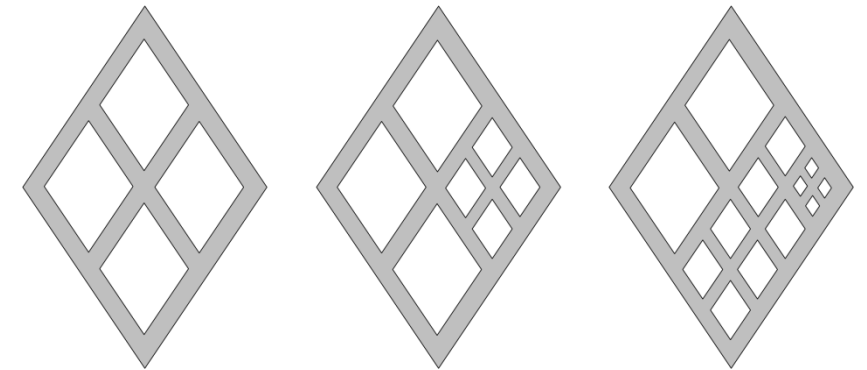
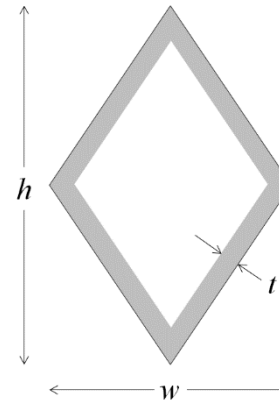
With infill,  
Unbalanced

# The Idea

- Rhombic cell: to ensure self-supporting
- Adaptive subdivision: as design variable in optimization

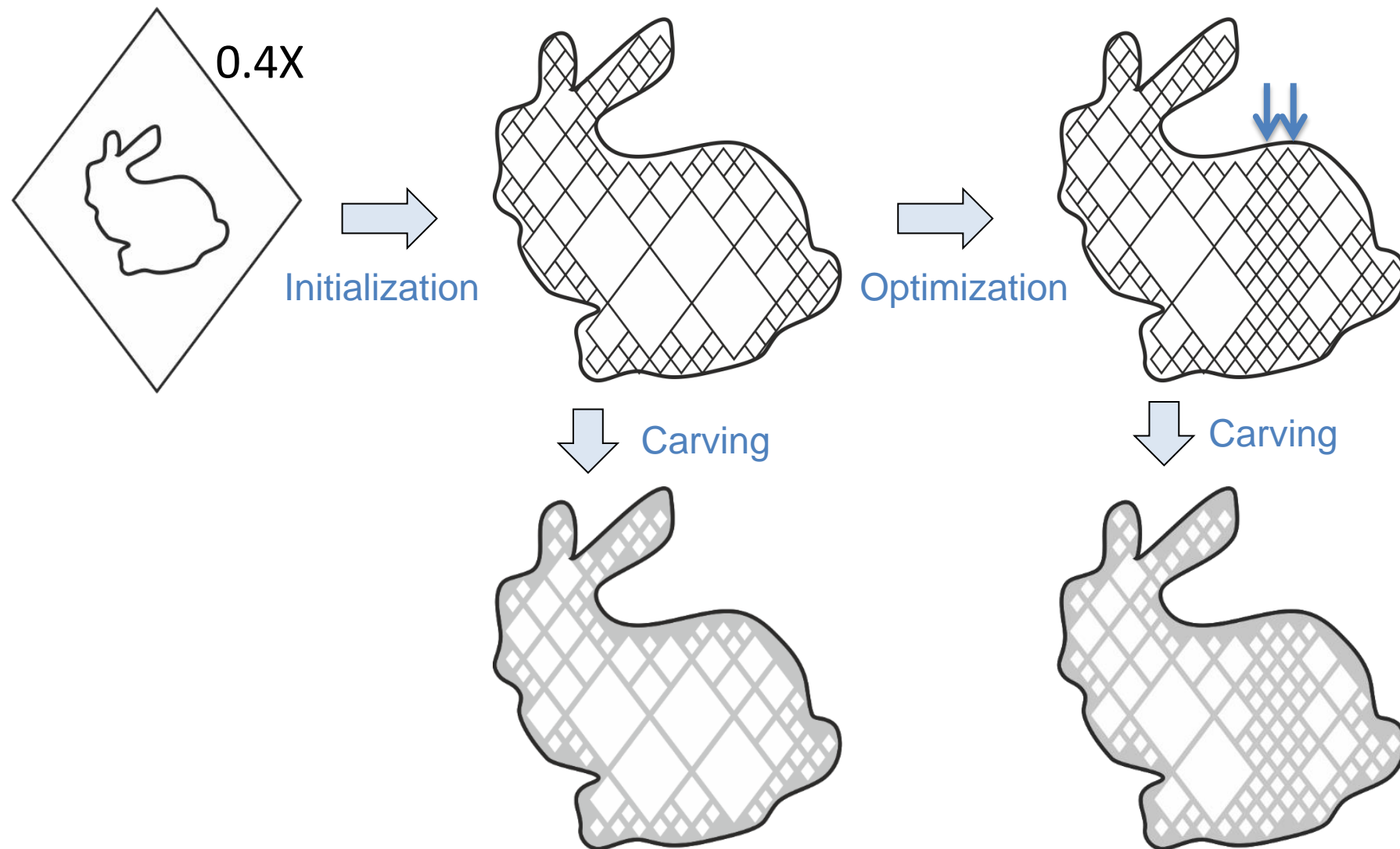


Rhombic cell



Adaptive subdivision

# Self-Supporting Rhombic Infill: Workflow



# Self-Supporting Rhombic Infill: Subdivision Criteria

- Min:  $c = \frac{1}{2} U^T K U$       Subject to:  $KU = F; V = \sum_i \rho_i \leq V_0$

**Voxel-wise** topology optimization

Per-voxel density as variable

$$\rho_i \in \{0.0, 1.0\}, \forall i$$

Per-voxel sensitivity:  $G_i = -\frac{\partial c / \partial \rho_i}{\partial V / \partial \rho_i}$

**Subdivision-based** topology optimization

Per-subdivision as variable

$$\beta_c \in \{0, 1\}, \forall c$$

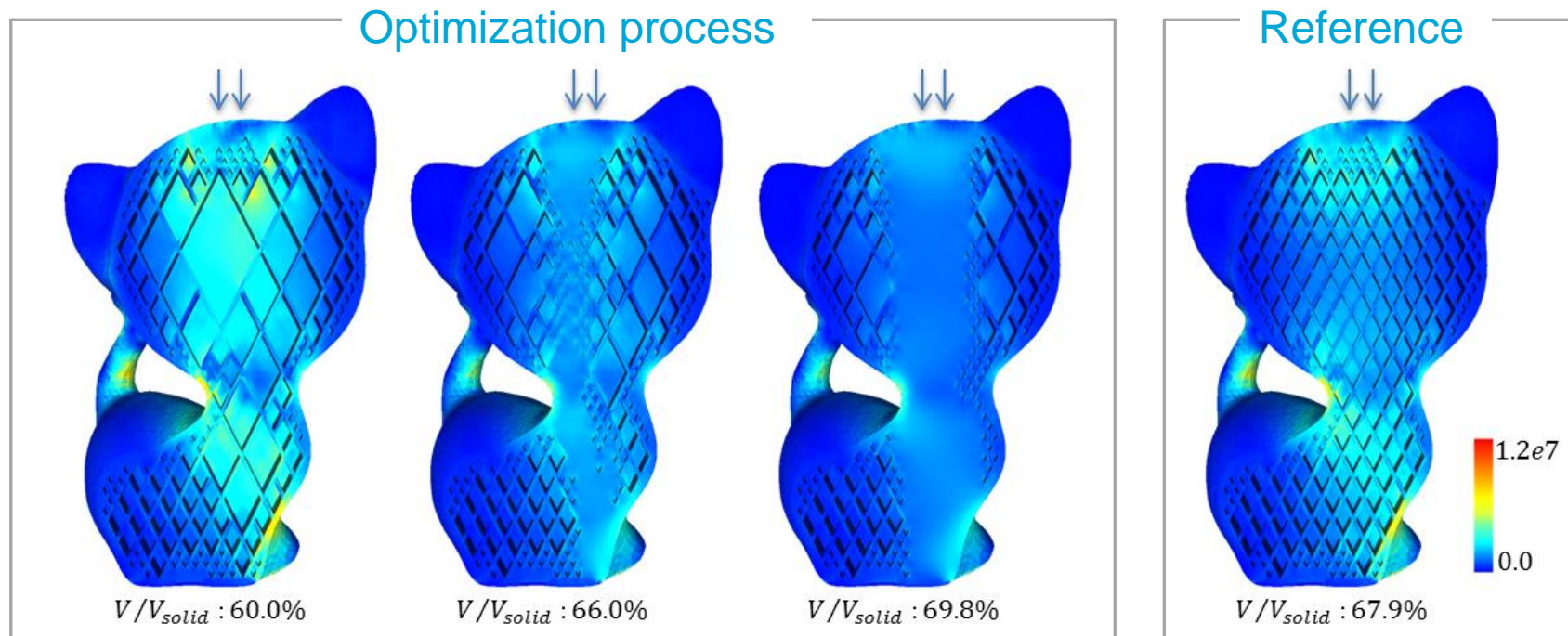
Per-voxel density assigned by subdivision

$$\rho_i(\beta) = \begin{cases} 1.0 & i \text{ covered by walls} \\ 0.0 & \text{otherwise} \end{cases}$$

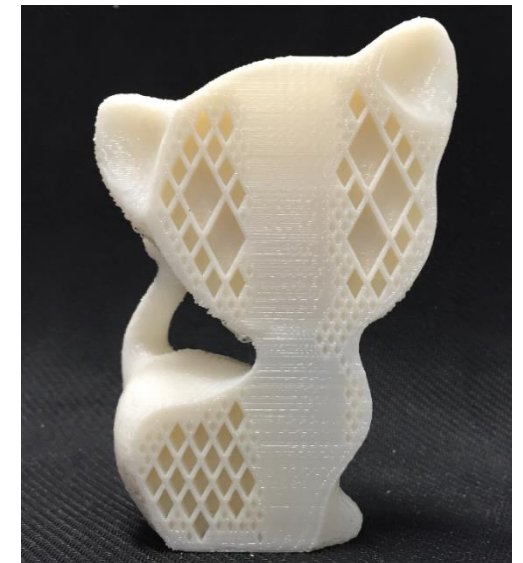
Per-subdivision sensitivity:  $G_c = -\frac{\partial c / \partial \beta_c}{\partial V / \partial \beta_c}$

# Self-Supporting Rhombic Infill: Results

- Optimized mechanical properties, compared to regular infill
- No additional inner supports needed

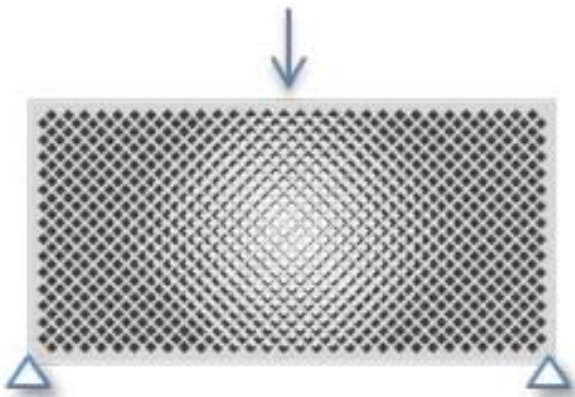
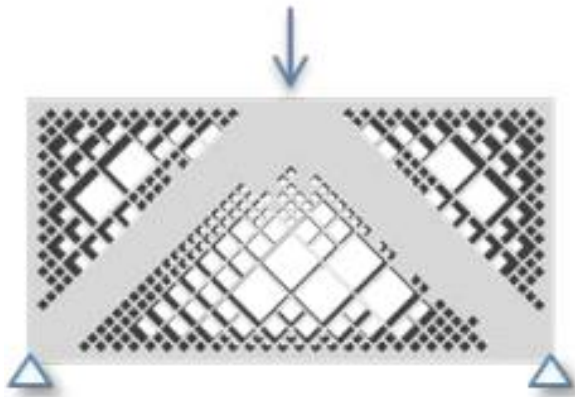


Print





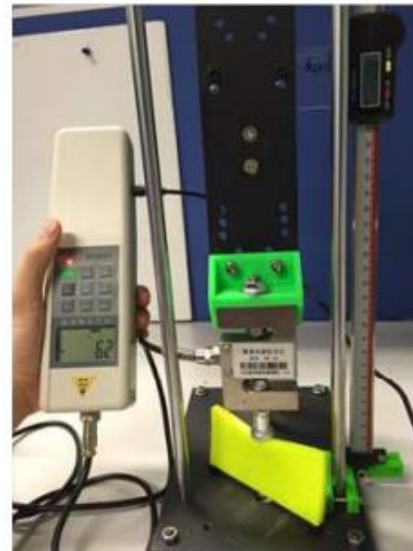
# Mechanical Tests



Under same force (62 N)



Dis.  
2.11 mm

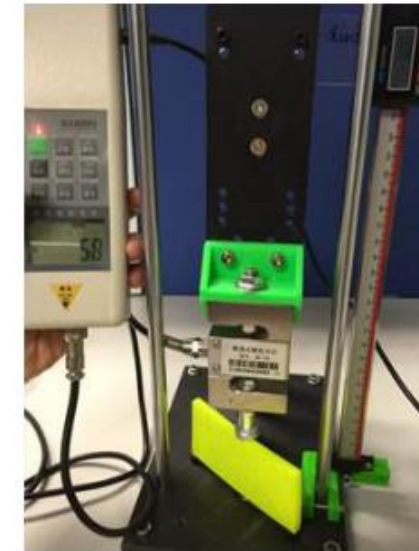


Dis.  
4.08 mm

Under same displacement (3.0 mm)



Force  
90 N



Force  
58 N

# Summary

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**



Thank you for your attention!

Questions?

Dr. Jun Wu

[j.wu-1@tudelft.nl](mailto:j.wu-1@tudelft.nl)

Depart. of Design Engineering, TU Delft



# Incomplete references: Density filters

- Guest, James K., Jean H. Prévost, and T. Belytschko. "[Achieving minimum length scale in topology optimization using nodal design variables and projection functions.](#)" International journal for numerical methods in engineering 61, no. 2 (2004): 238-254.
- Wang, Fengwen, Boyan Stefanov Lazarov, and Ole Sigmund. "[On projection methods, convergence and robust formulations in topology optimization.](#)" Structural and Multidisciplinary Optimization 43, no. 6 (2011): 767-784.
- Clausen, Anders, Niels Aage, and Ole Sigmund. "[Topology optimization of coated structures and material interface problems.](#)" Computer Methods in Applied Mechanics and Engineering 290 (2015): 524-541.
- Langelaar, Matthijs. "[An additive manufacturing filter for topology optimization of print-ready designs.](#)" Structural and Multidisciplinary Optimization (2016): 1-13.
- Wu, Jun, Niels Aage, Ruediger Westermann, and Ole Sigmund. "[Infill Optimization for Additive Manufacturing--Approaching Bone-like Porous Structures.](#)" IEEE Transactions on Visualization and Computer Graphics, 2016.

# Incomplete references: Alternative parameterizations

- Wang, Weiming, Tuanfeng Y. Wang, Zhouwang Yang, Ligang Liu, Xin Tong, Weihua Tong, Jiansong Deng, Falai Chen, and Xiuping Liu. "[Cost-effective printing of 3D objects with skin-frame structures](#)." ACM Transactions on Graphics (TOG) 32, no. 6 (2013): 177.
- Lu, Lin, Andrei Sharf, Haisen Zhao, Yuan Wei, Qingnan Fan, Xuelin Chen, Yann Savoye, Changhe Tu, Daniel Cohen-Or, and Baoquan Chen. "[Build-to-last: Strength to weight 3d printed objects](#)." ACM Transactions on Graphics (TOG) 33, no. 4 (2014): 97.
- Musialski, Przemyslaw, Thomas Auzinger, Michael Birsak, Michael Wimmer, and Leif Kobbelt. "[Reduced-order shape optimization using offset surfaces](#)." ACM Trans. Graph. 34, no. 4 (2015): 102.
- Wu, Jun, Lou Kramer, and Rüdiger Westermann. "[Shape interior modeling and mass property optimization using ray-reps](#)." Computers & Graphics 58 (2016): 66-72.
- Wu, Jun, Charlie CL Wang, Xiaoting Zhang, and Rüdiger Westermann. "[Self-supporting rhombic infill structures for additive manufacturing](#)." Computer-Aided Design 80 (2016): 32-42.