EUROGRAPHICS 2017
The 38th annual conference of the
EUROPEAN ASSOCIATION FOR COMPUTER GRAPHICS

Topology Optimization for Computational Fabrication

Jun Wu, Niels Aage, Sylvain Lefebvre, Charlie Wang

TU Delft
DTU Technical University of Denmark
Inria INVENTEURS DU MONDE NUMÉRIQUE

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Topology Optimization for Computational Fabrication

Part 4: Topology Optimization for Appearance and Structure Synthesis

Sylvain Lefebvre
Inria

Textures in Computer Graphics

Authoring textures

Forza Horizon 3, Microsoft Studios *https://www.forzamotorsport.net/en-us/games/fh3*

Authoring textures

Too much content to be done entirely manually

Forza Horizon 3, Microsoft Studios *https://www.forzamotorsport.net/en-us/games/fh3*

Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g. erosion)

We will see both in the context of fabrication

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  - By-example synthesis
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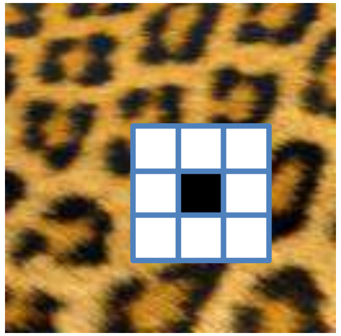
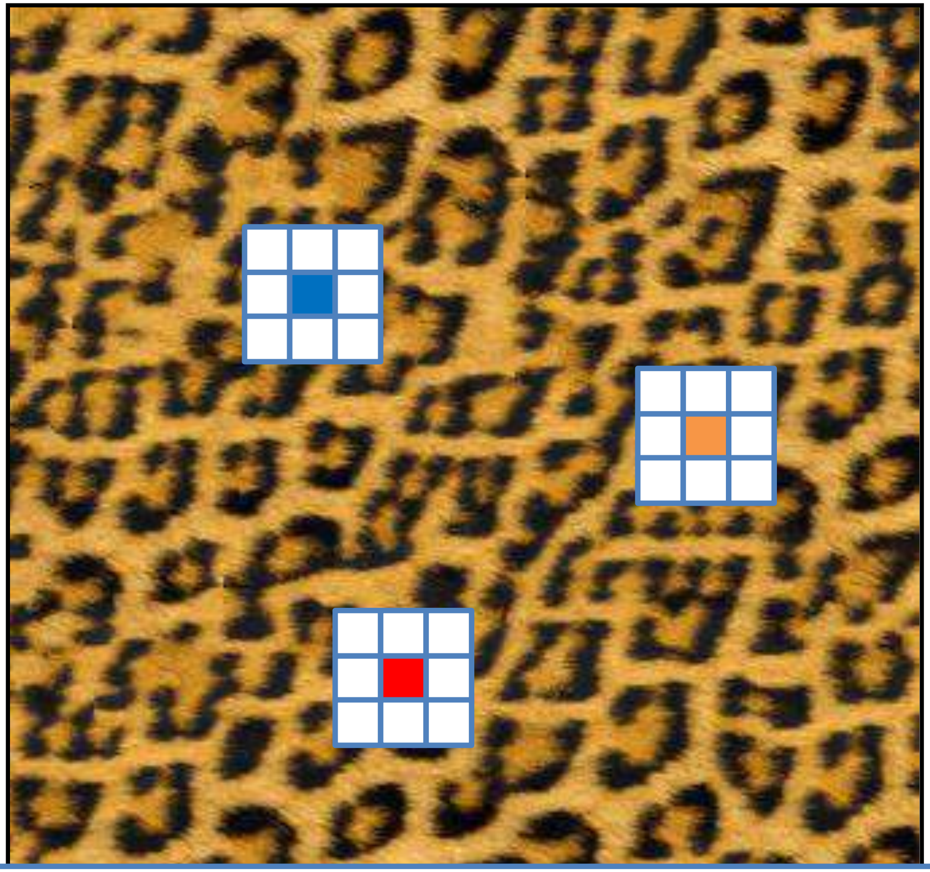
Texture synthesis: color formulation

(color field)

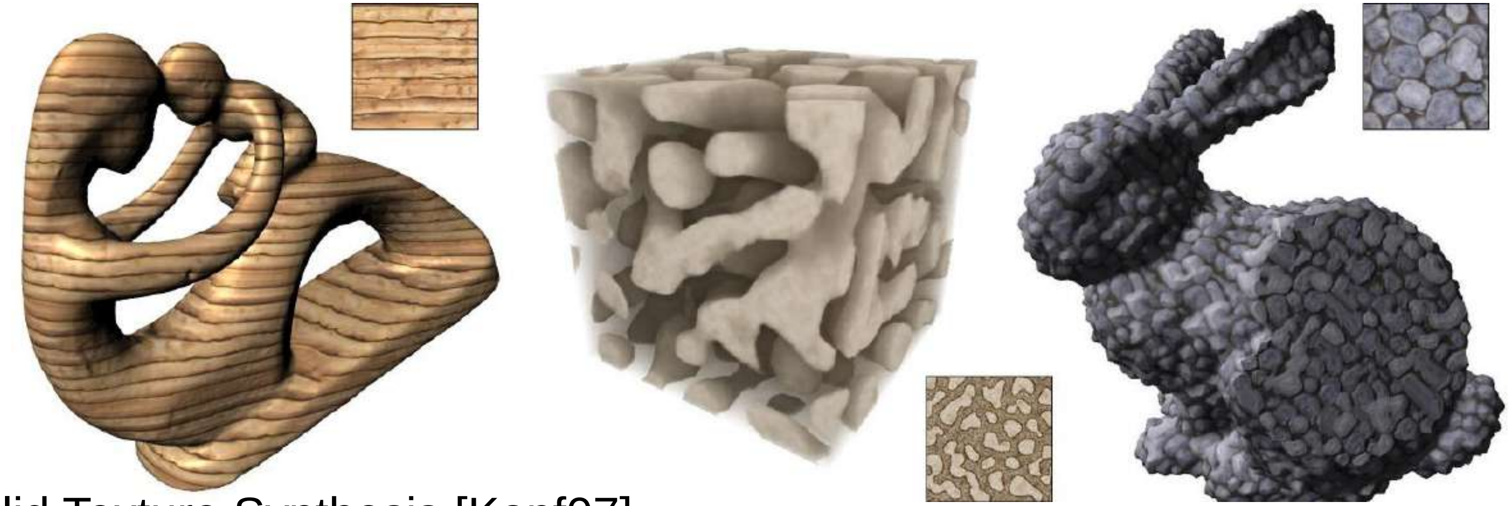


Exemplar

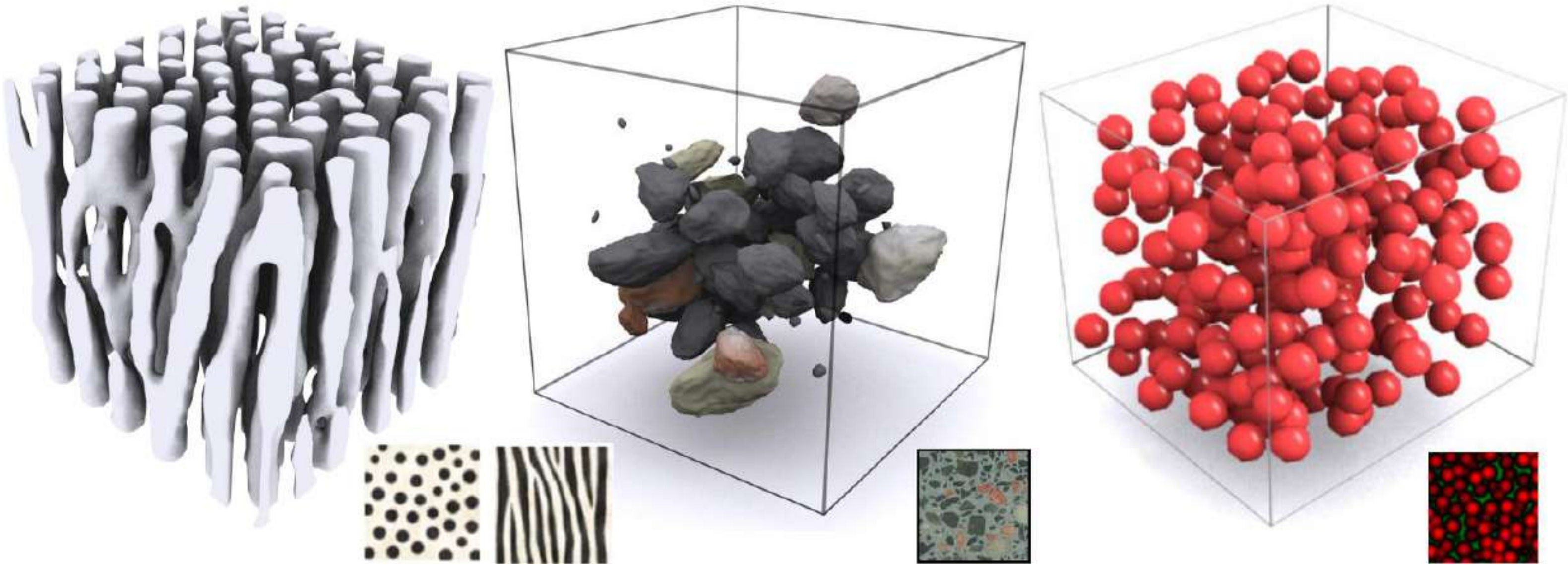


Assumption (MRF):
Same neighborhoods at all scales ➔ Same visual content

Volume Texture Synthesis

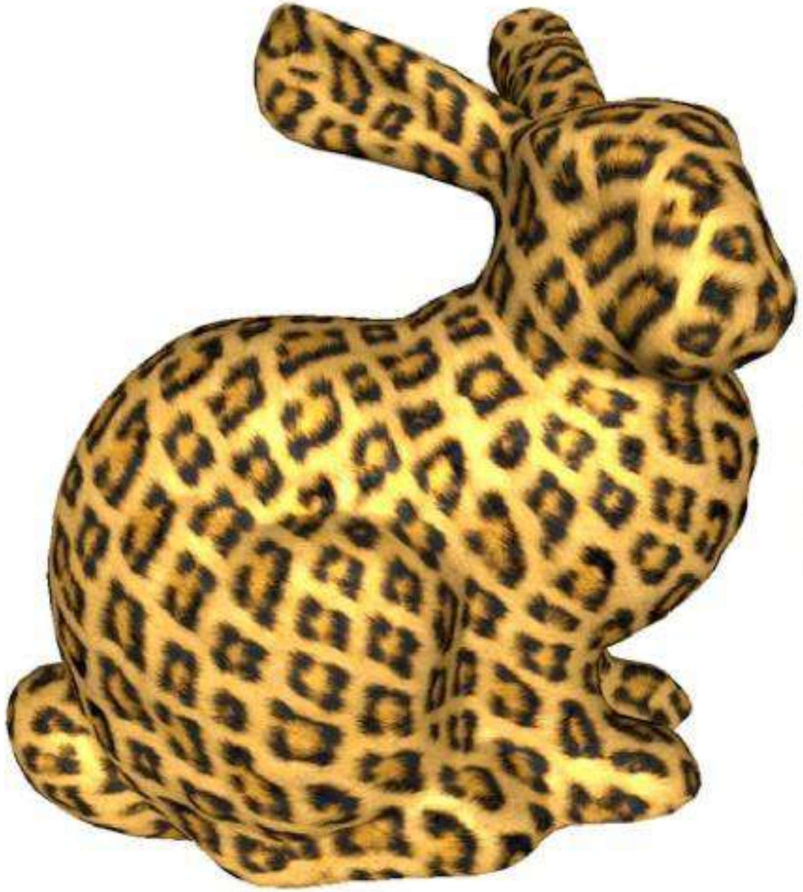


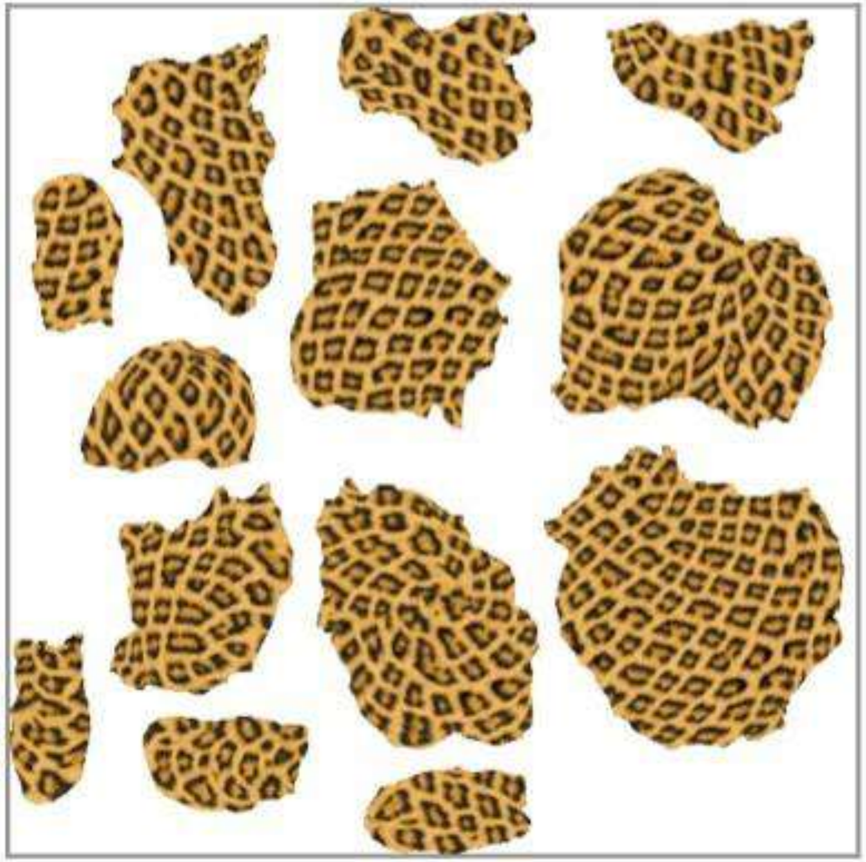
Solid Texture Synthesis [Kopf07]



Lazy Solid Texture Synthesis [Dong08]

On-surface texture synthesis

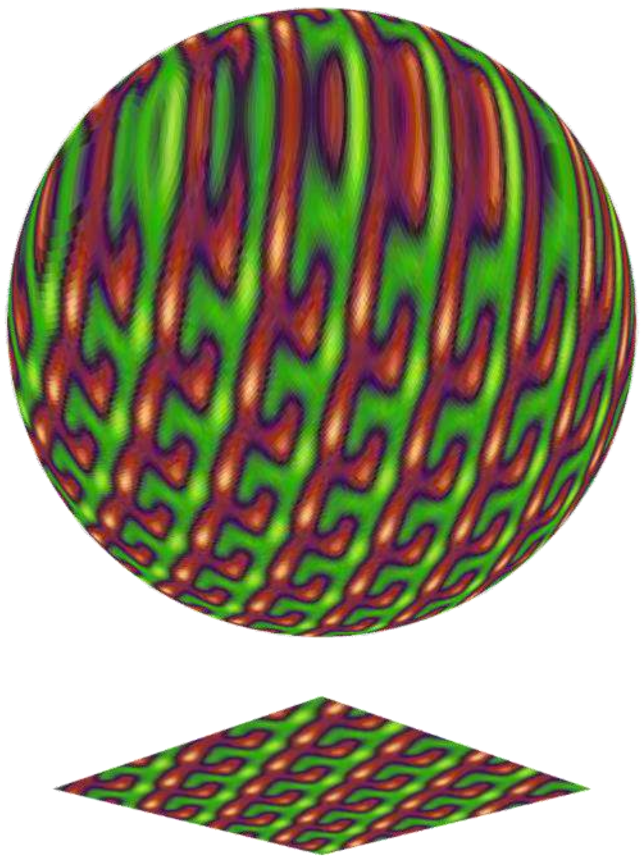




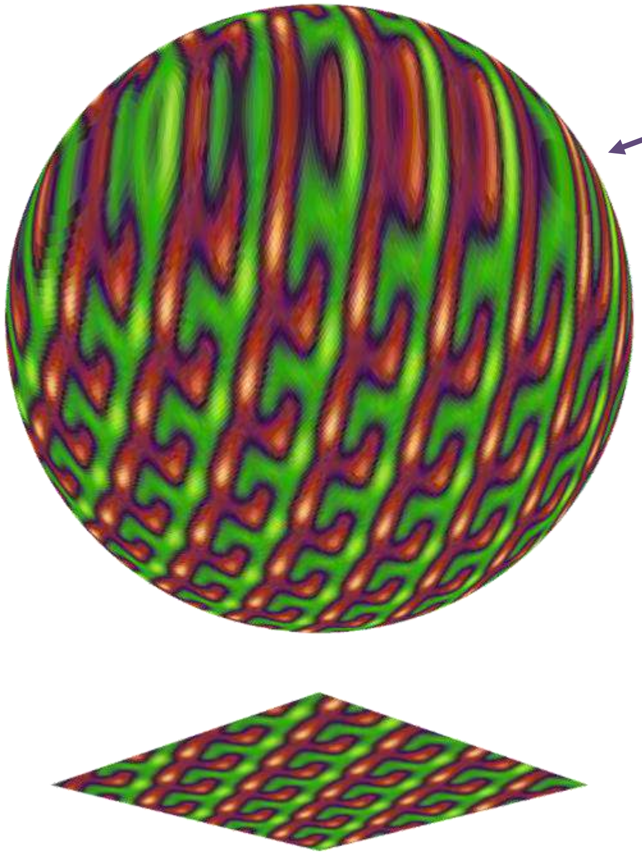
[Lefebvre and Hoppe 2006]

On-surface texture synthesis, the easier way

On-surface texture synthesis, the easier way

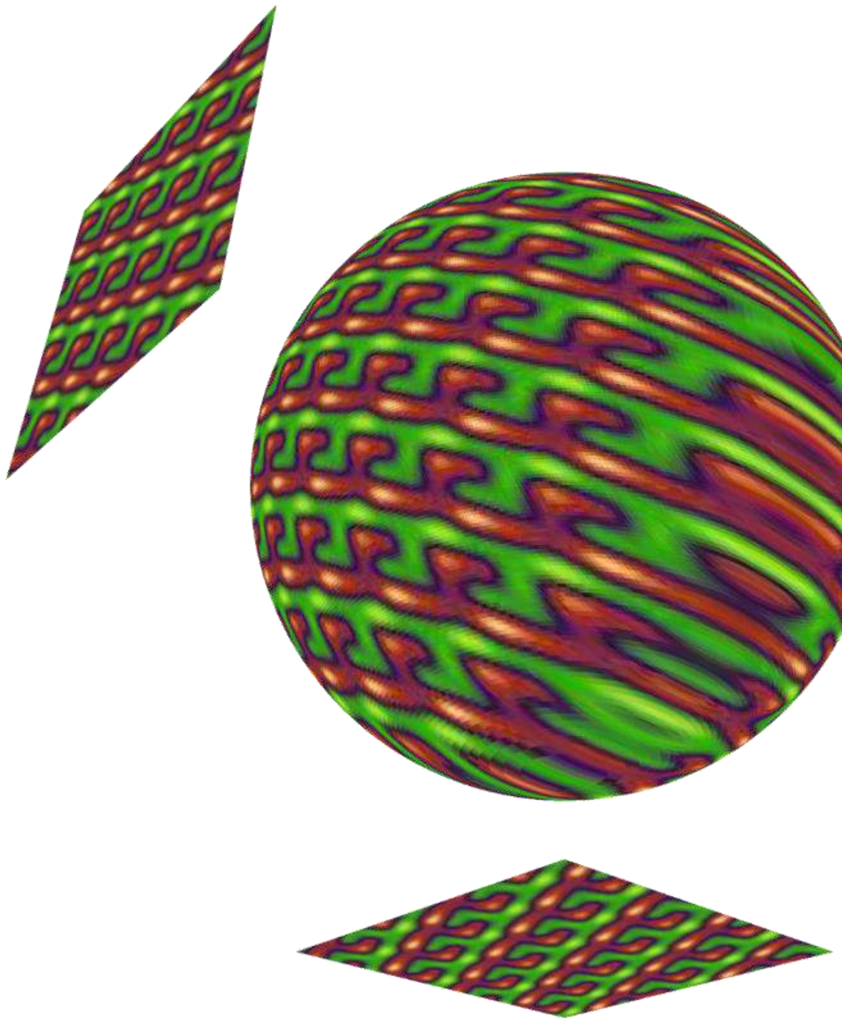


On-surface texture synthesis, the easier way



Distortion!

On-surface texture synthesis, the easier way



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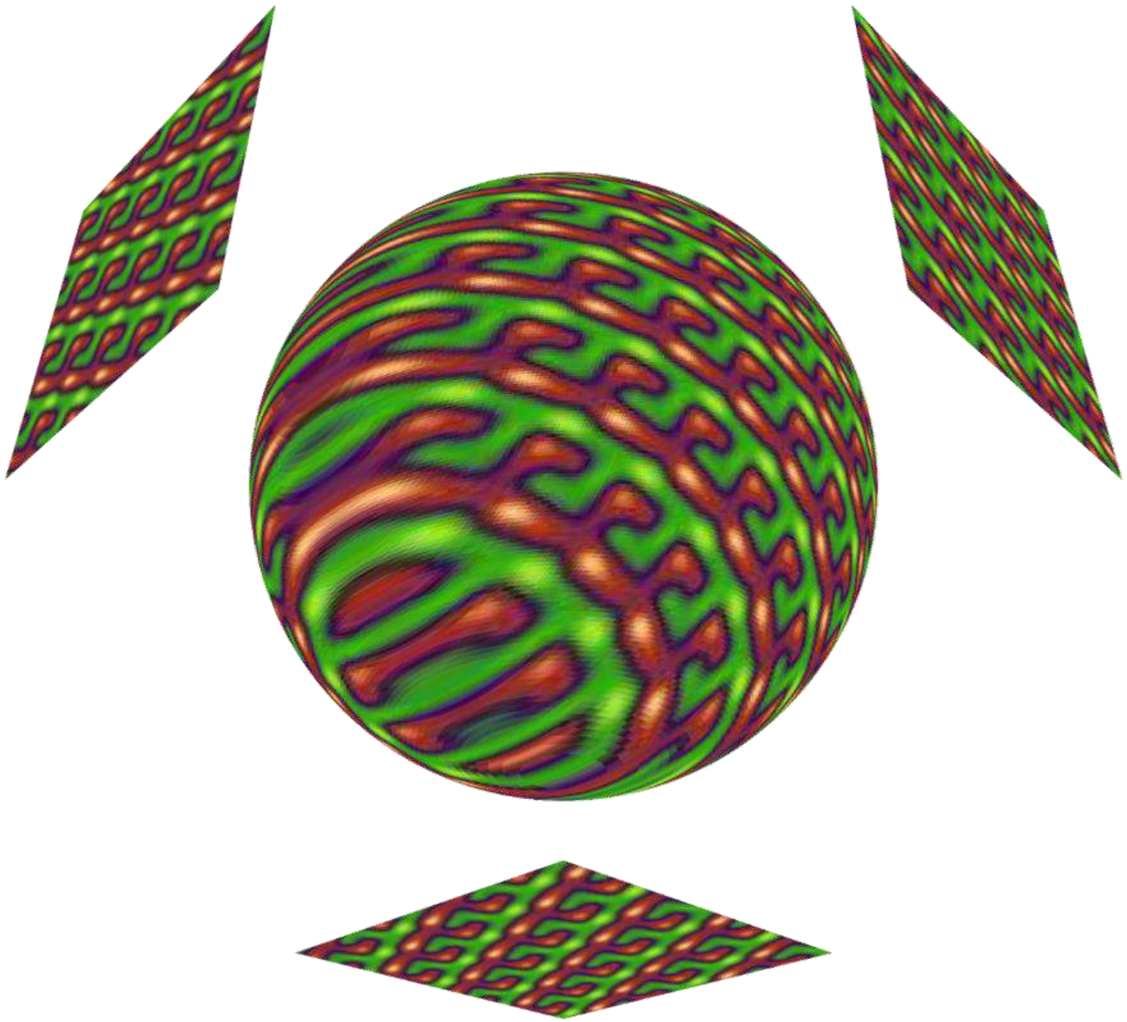
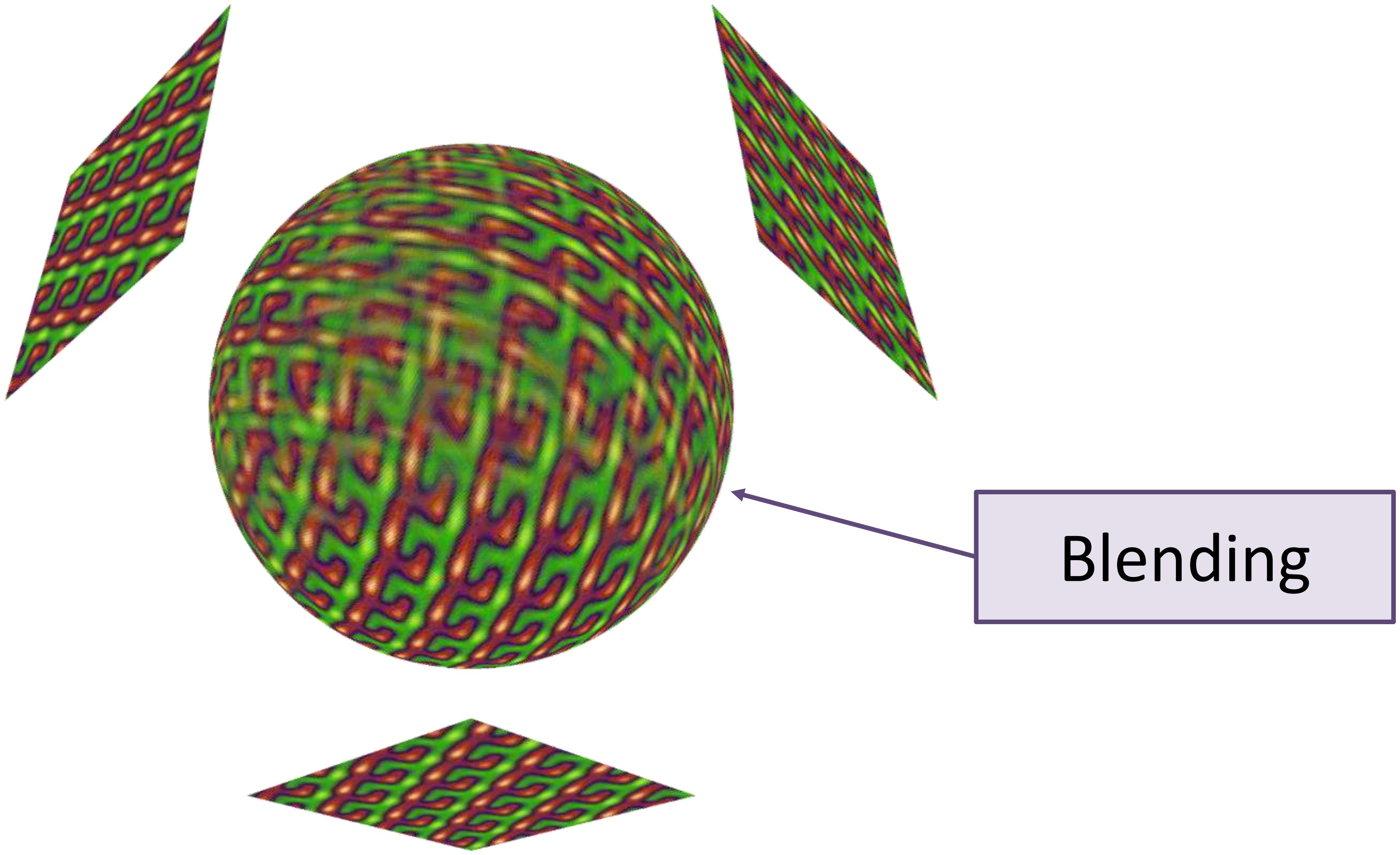


On-surface texture synthesis, the easier way



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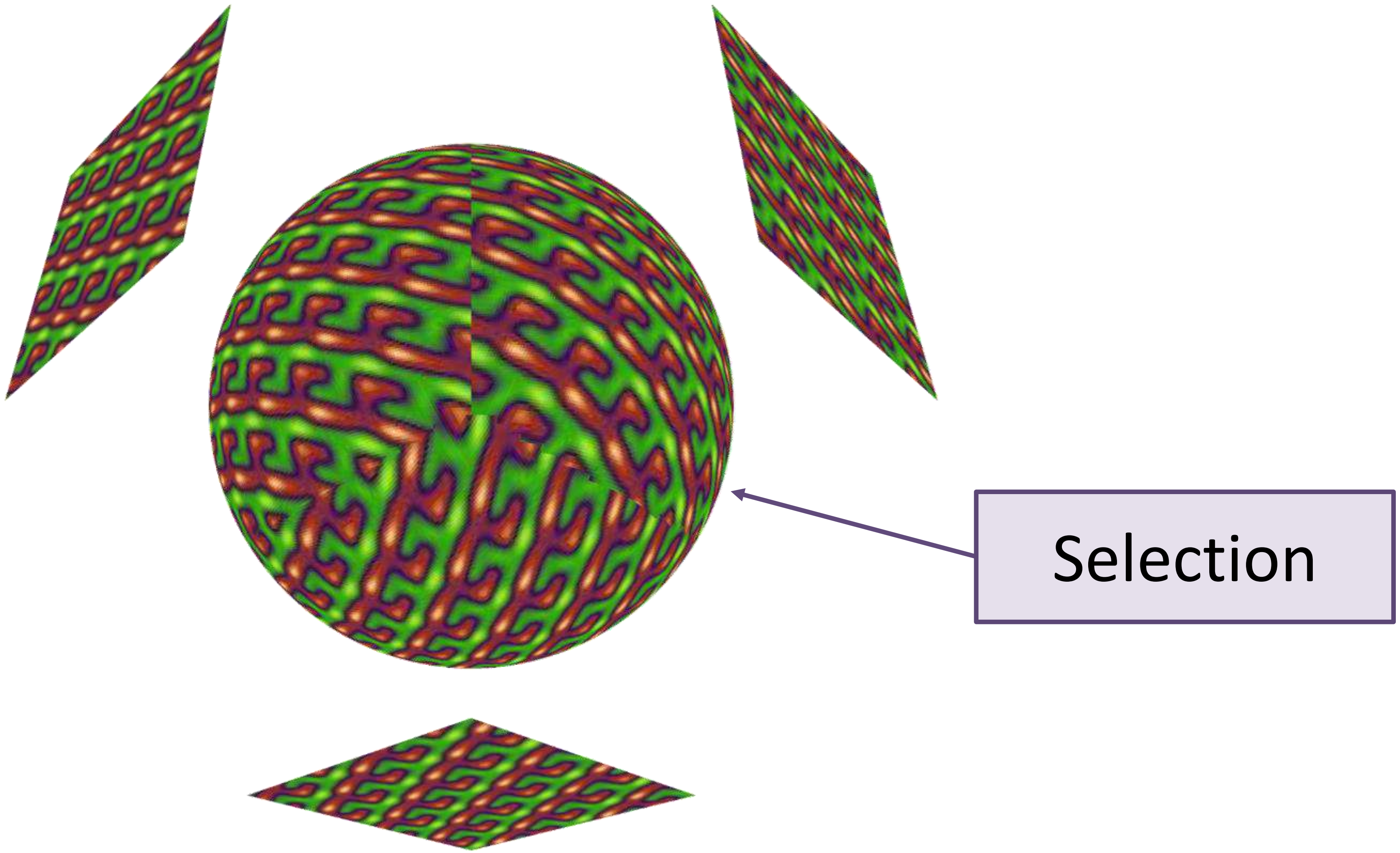

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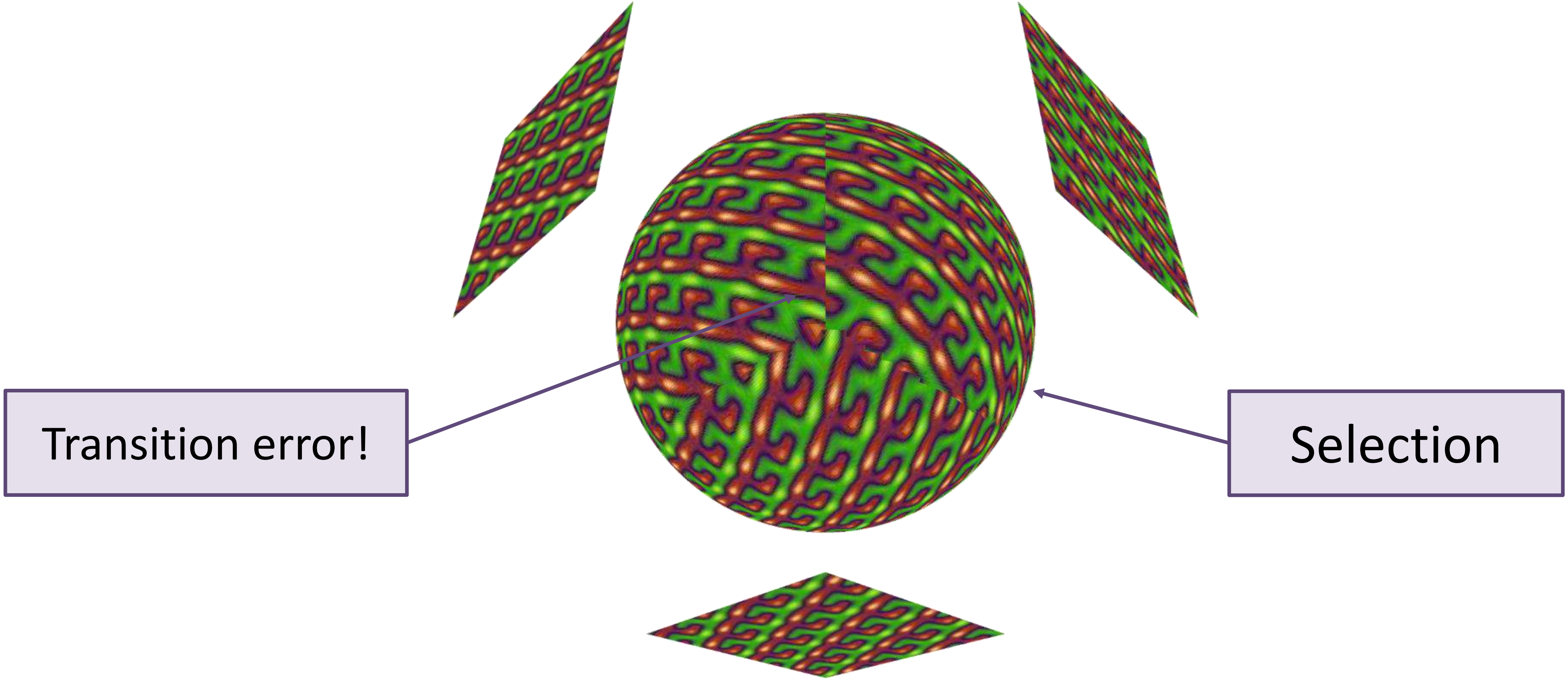

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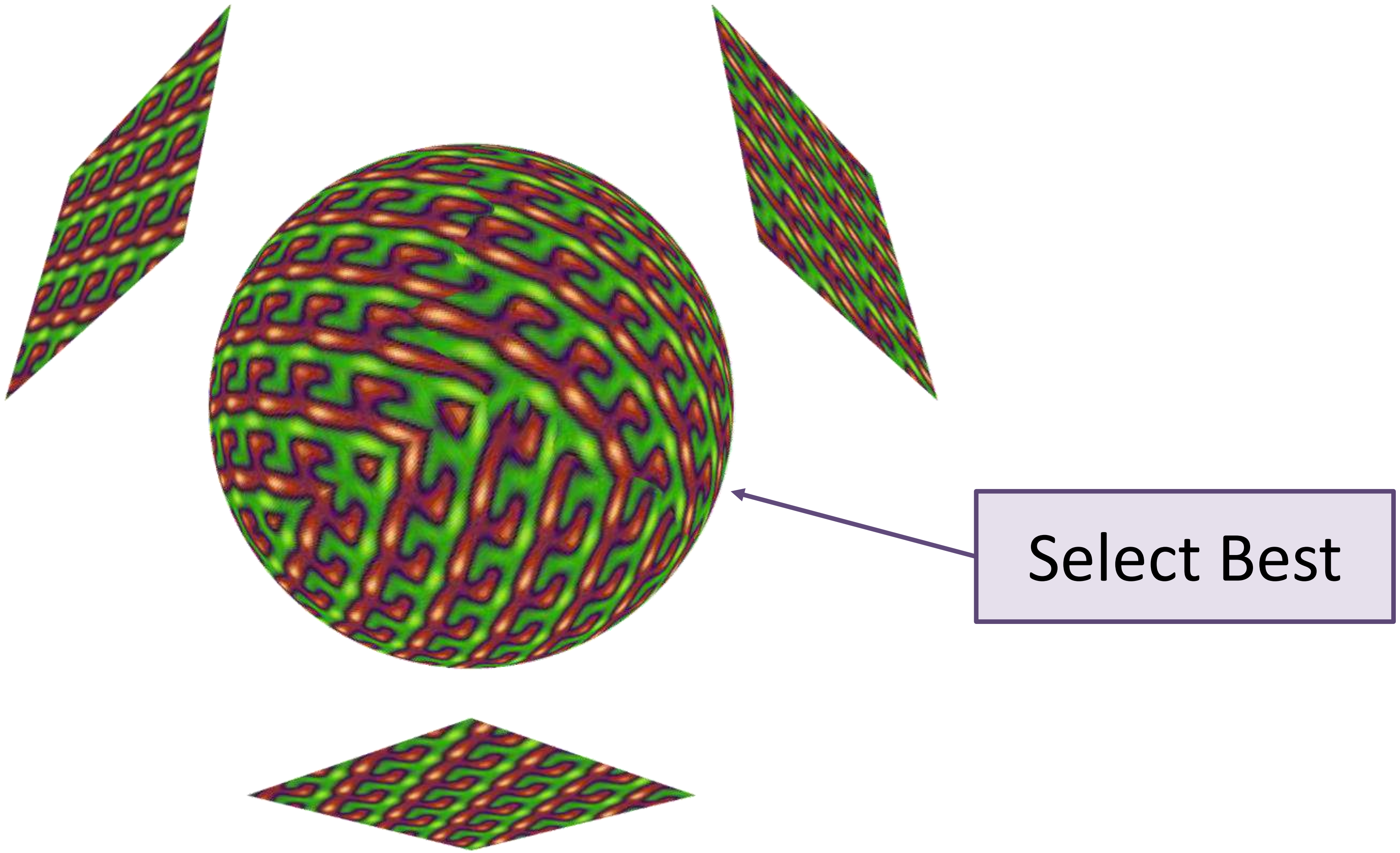

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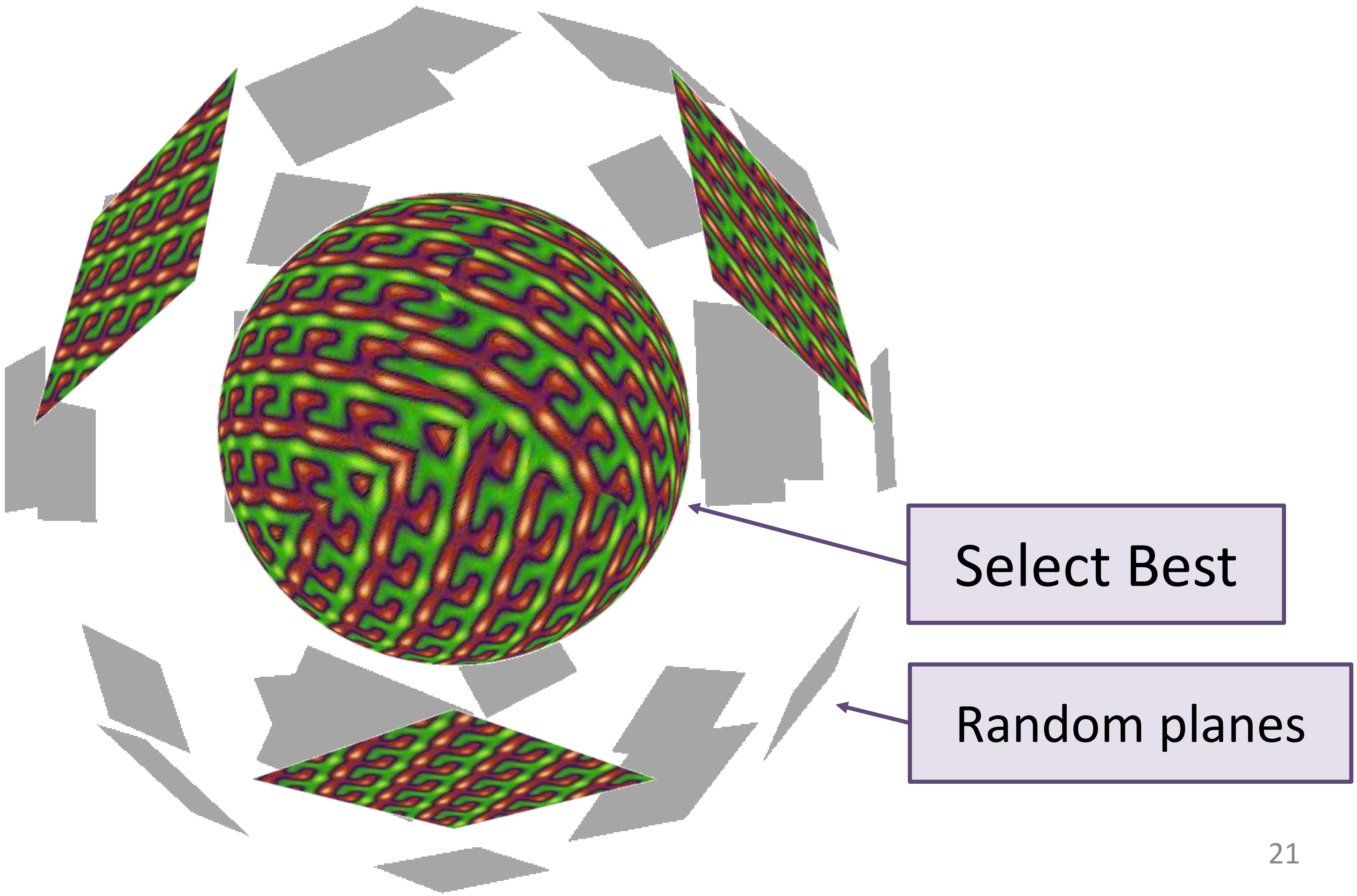

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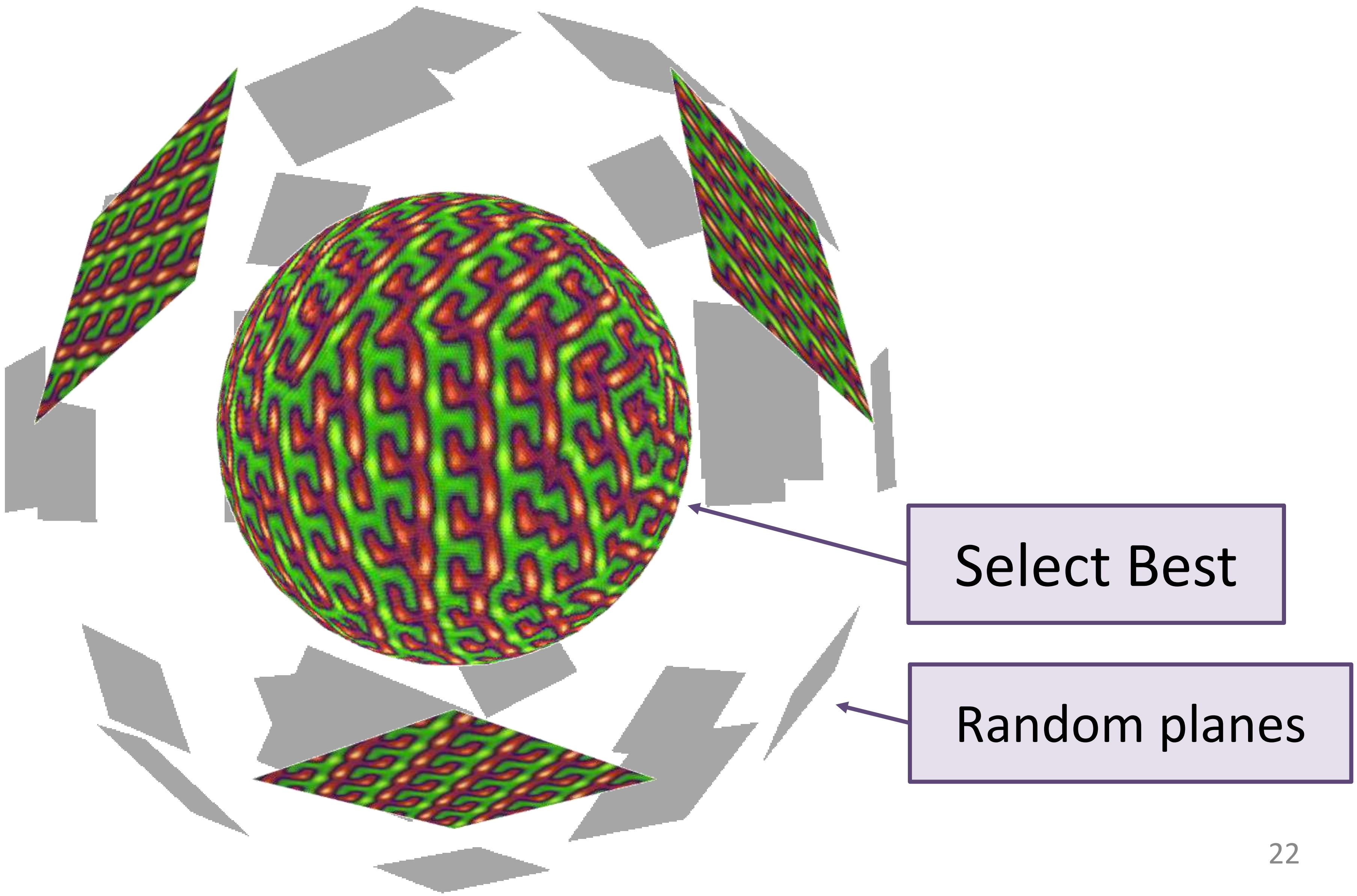

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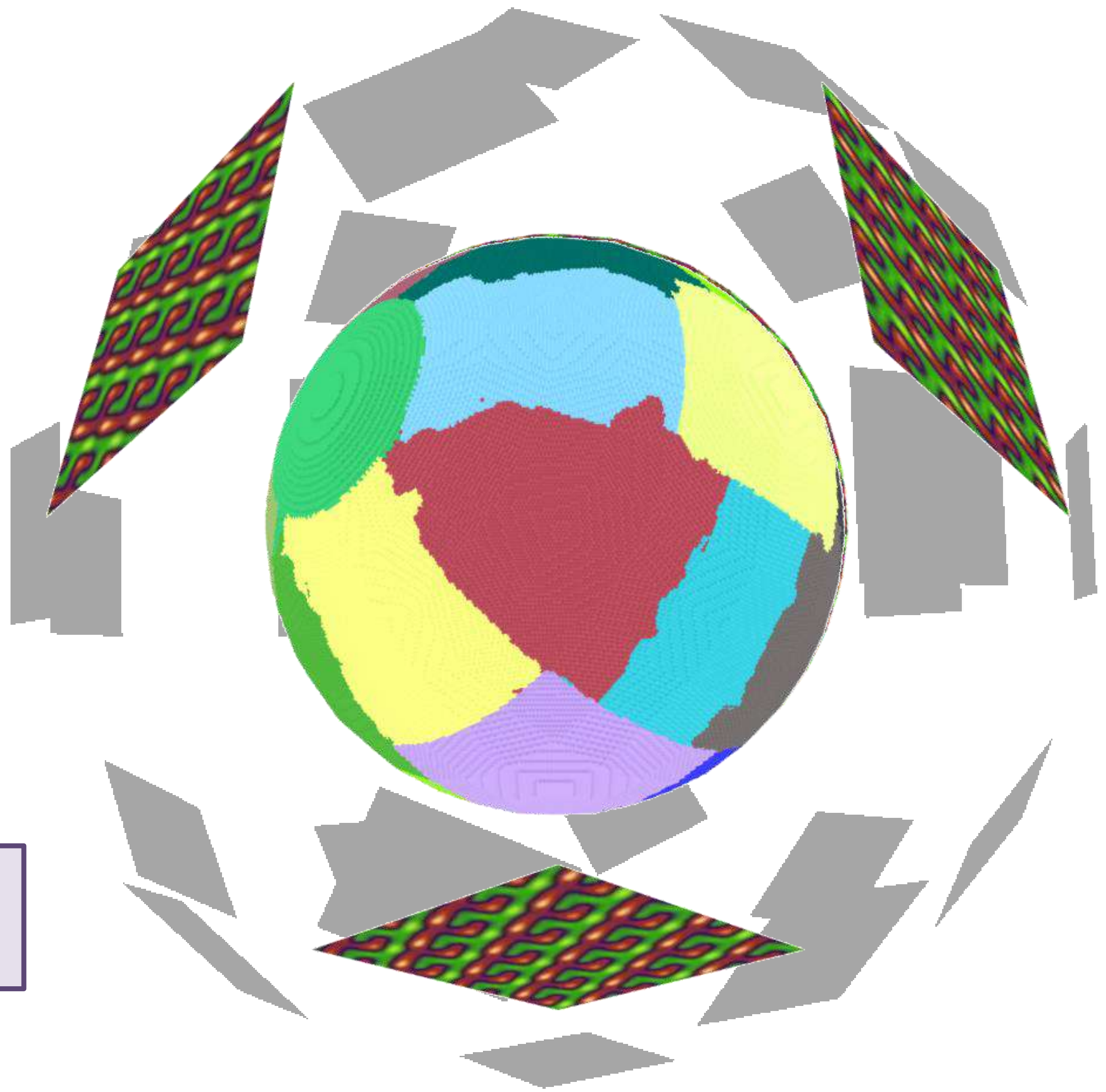
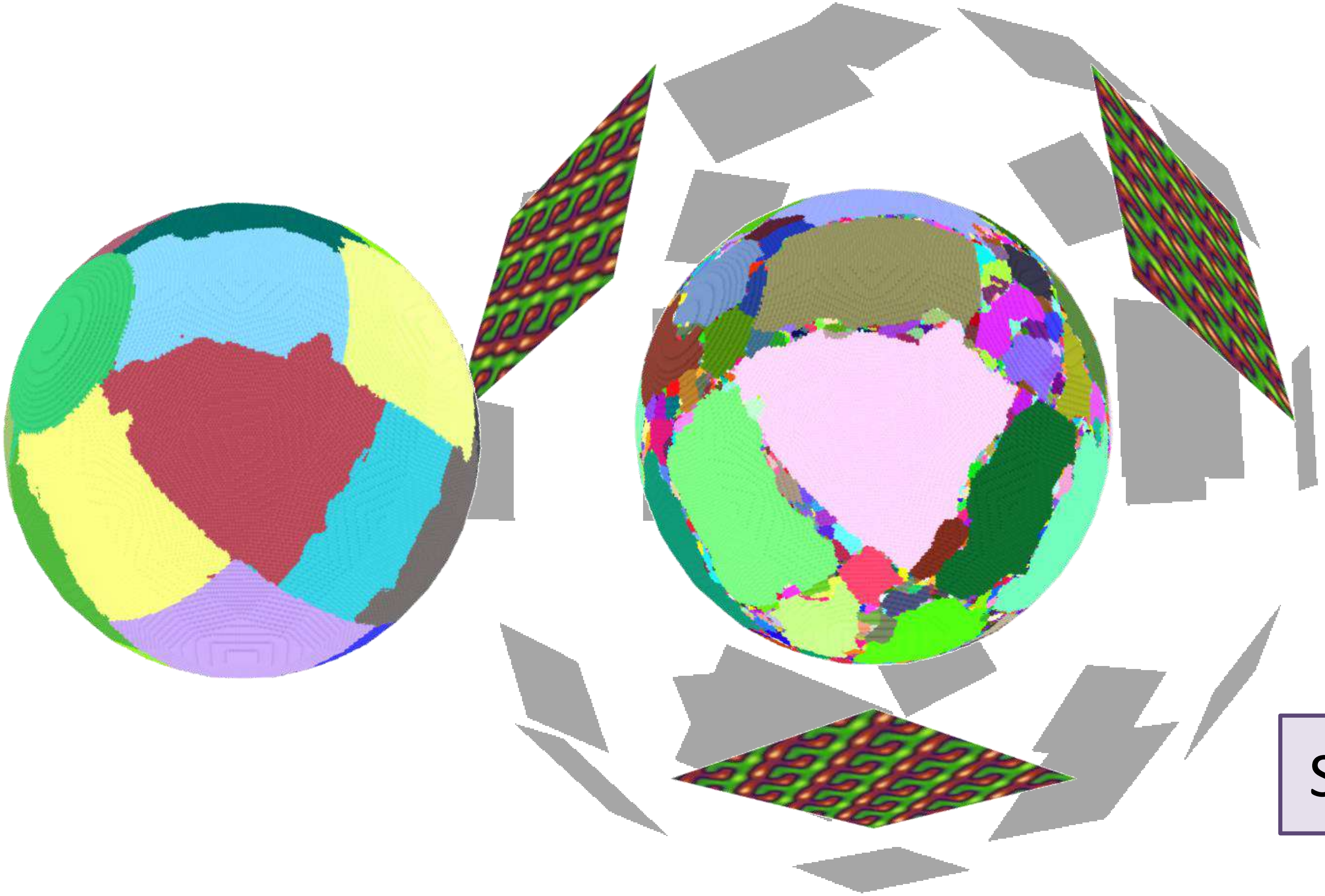


Plane choices

On-surface texture synthesis, the easier way



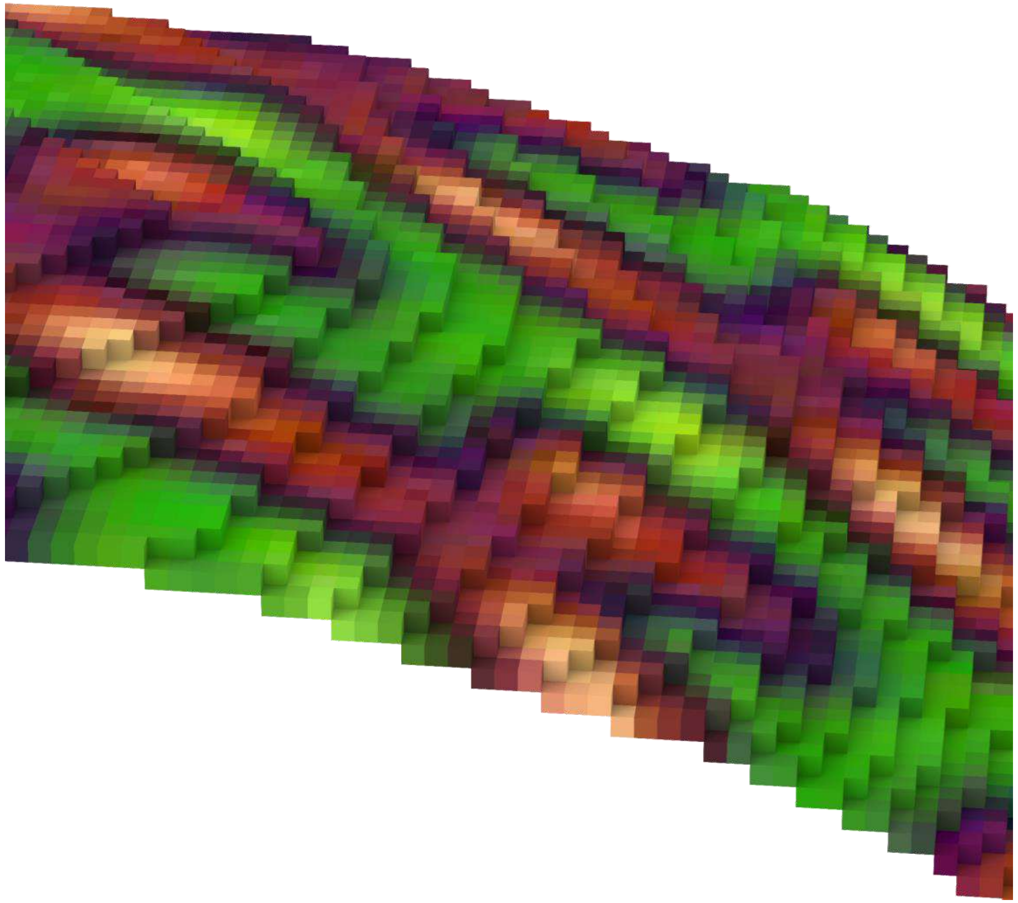
Shifts + Rotations

Labelling Problem

- Surface neighborhood (2D)

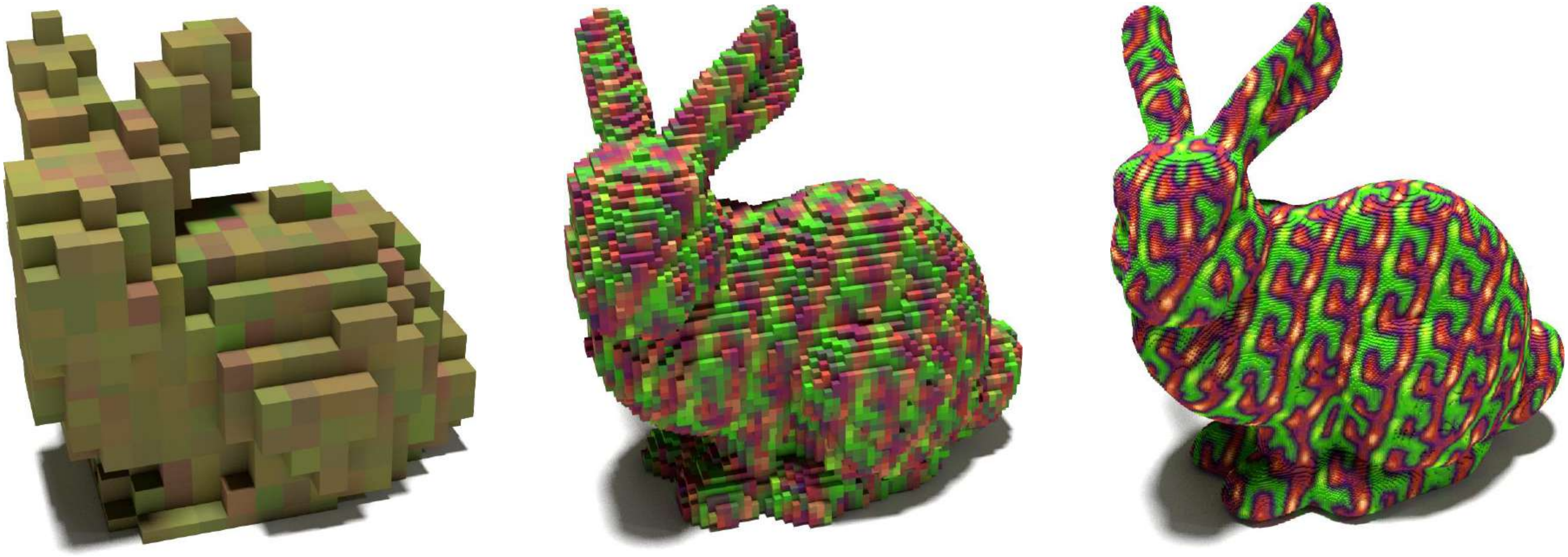
Transition error

Distortion error

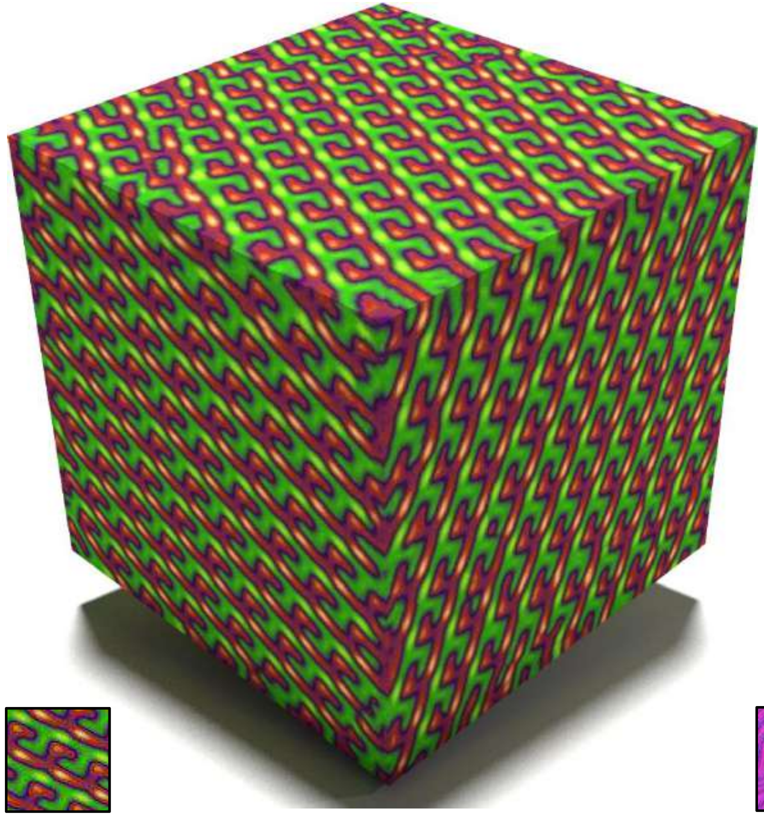


Multiresolution Synthesis

- Upsample, jitter, correction [Lefebvre and Hoppe 2005]



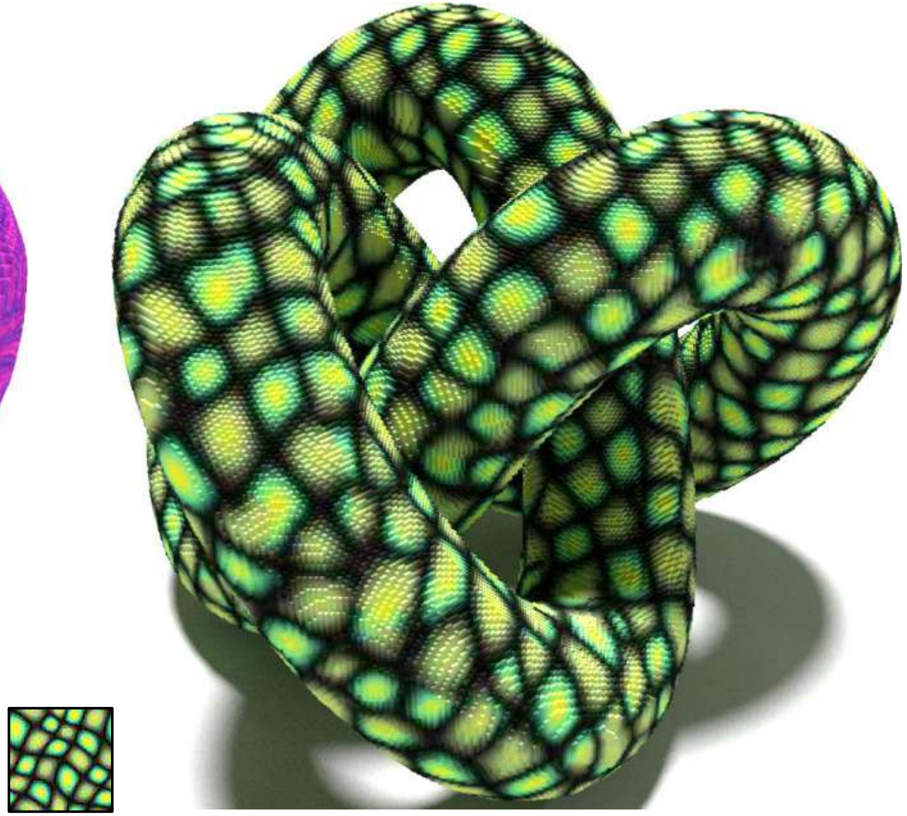
Results



Time 28.6s

Time 14.7s

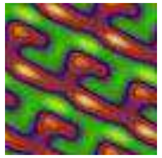
thing:168602 (Steelyd)

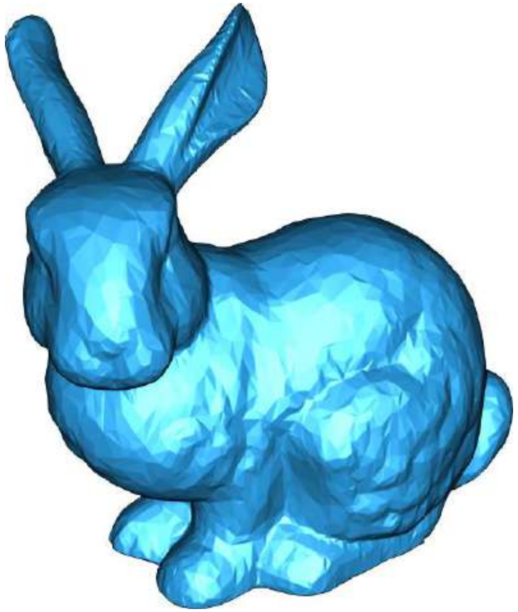


Time 18.7s

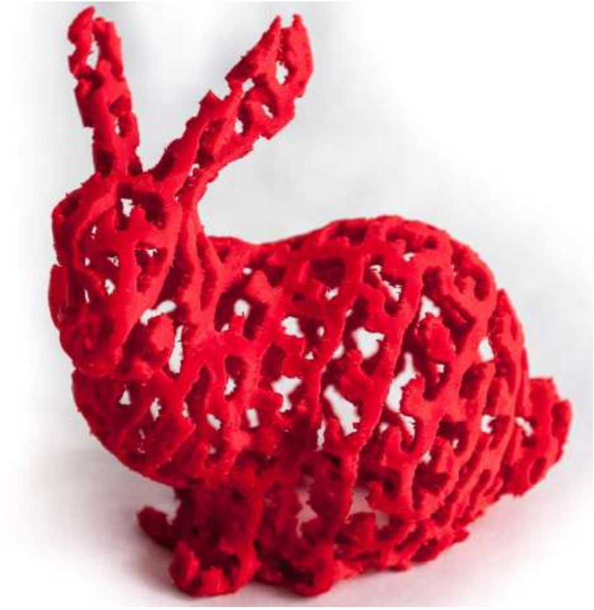
thing:5506 (chylld)

Texture as structure?

Model + appearance  + structure 



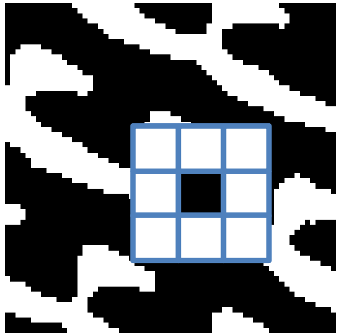
Texture Synthesis?



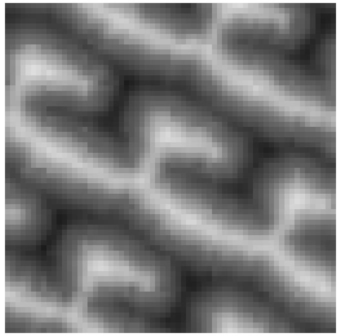
???

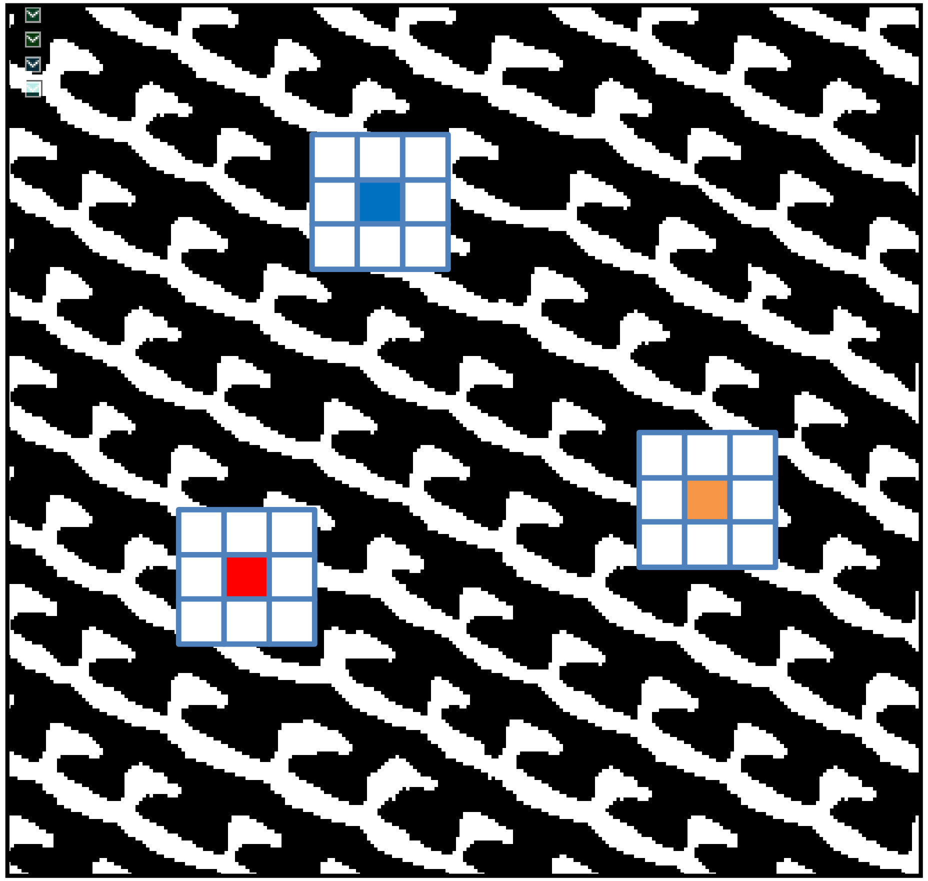
Texture synthesis: structure formulation

(density field)



Exemplar

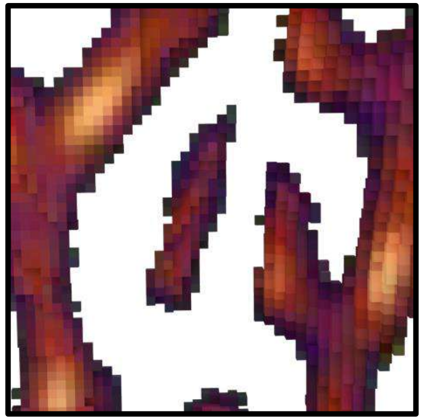


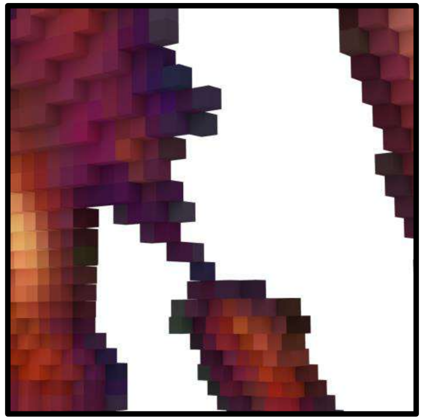


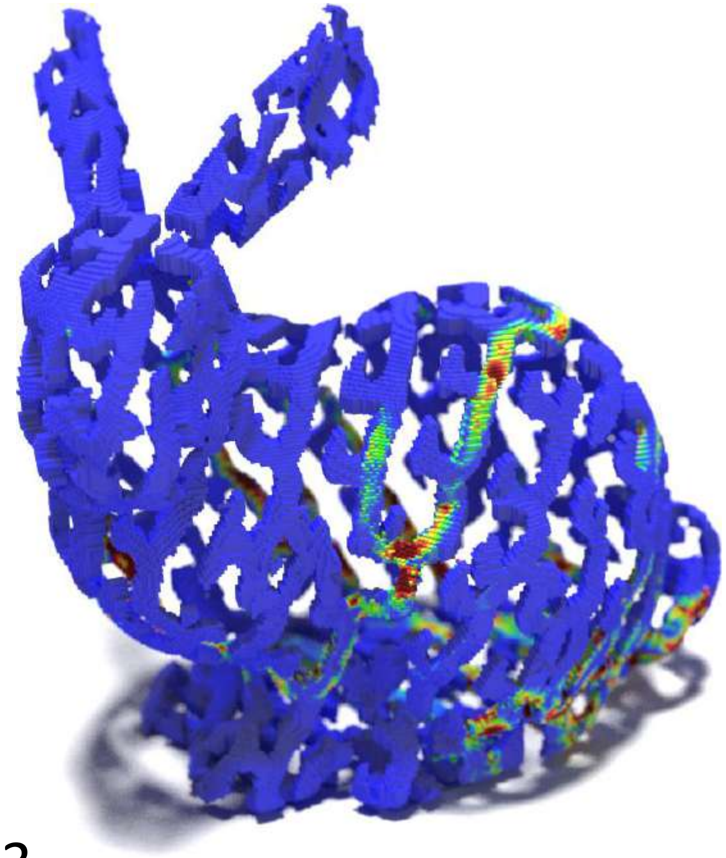
Neighborhoods capture ***local geometry*** accross scales

Printability

1. Connected components
2. Minimum thickness
3. No weak part (rigidity)

1. 

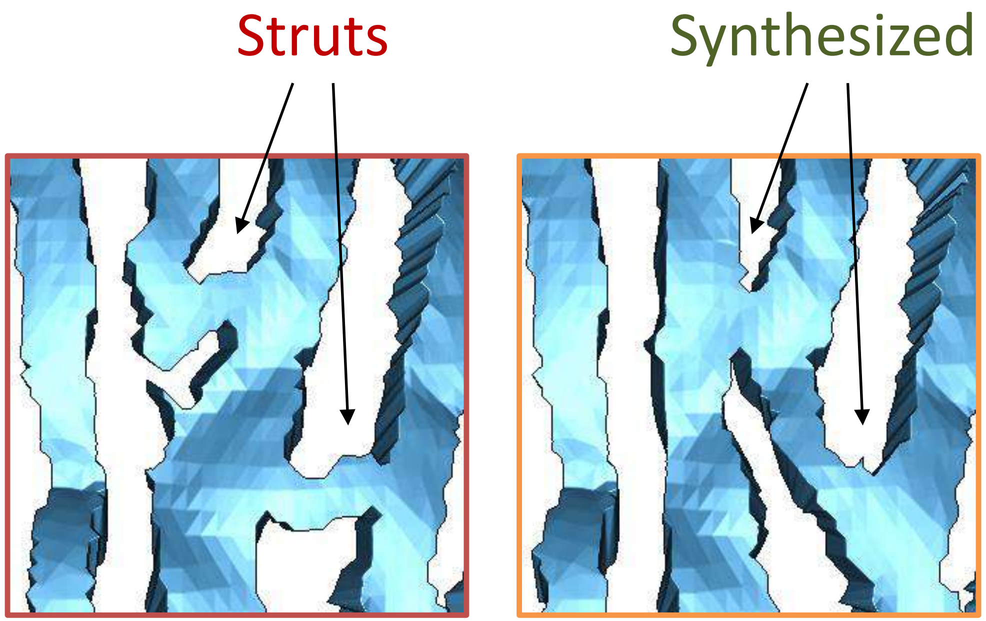
2. 

3. 

Key ideas for structure synthesis

Pattern is stochastic

- Exhibits degrees of freedom
- Use pattern itself to locally reinforce structure



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Exemplar specifies local geometry

- Large scale arrangement can be optimized ‘orthogonally’
- Combination with topology optimization?

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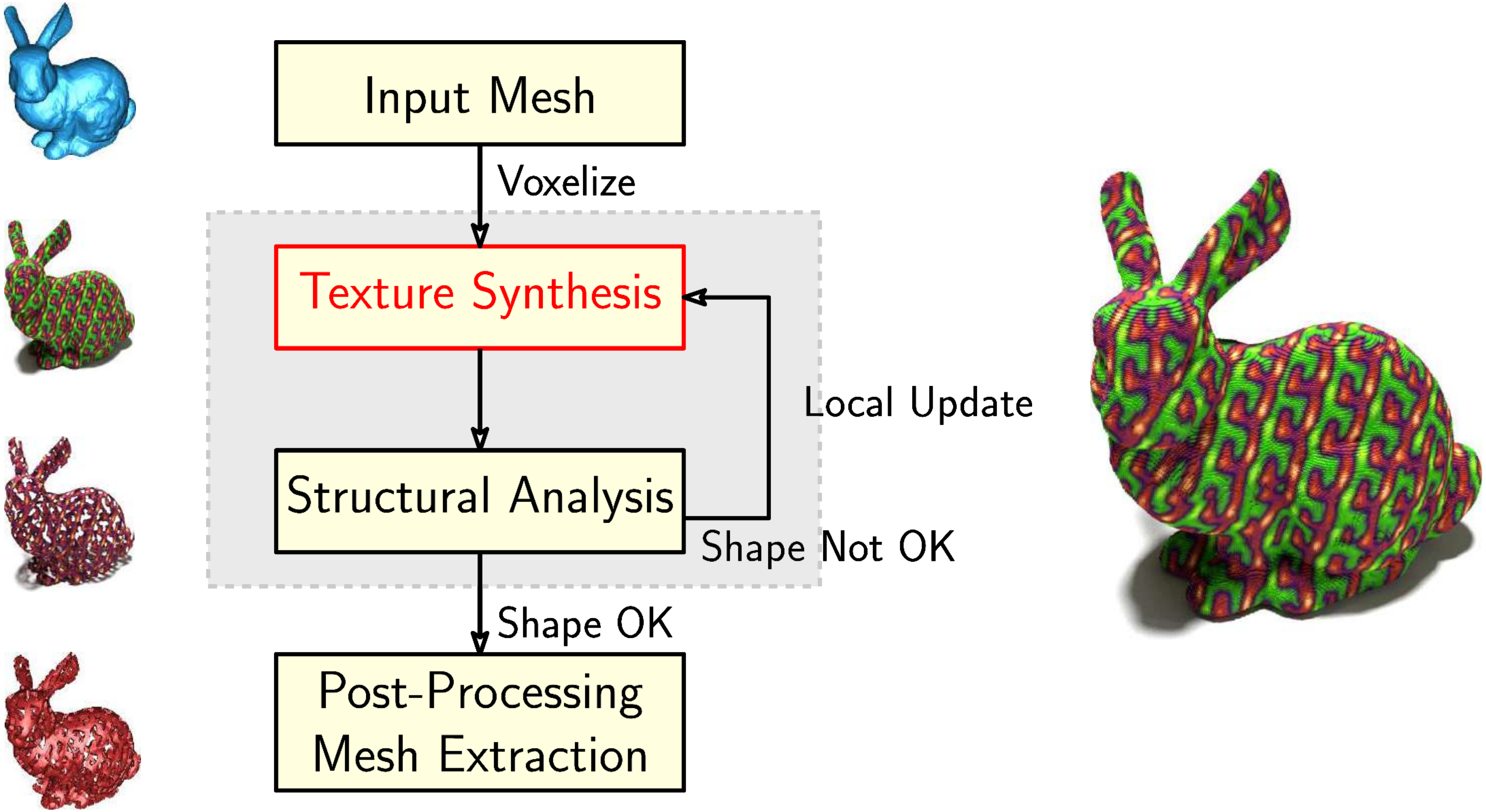
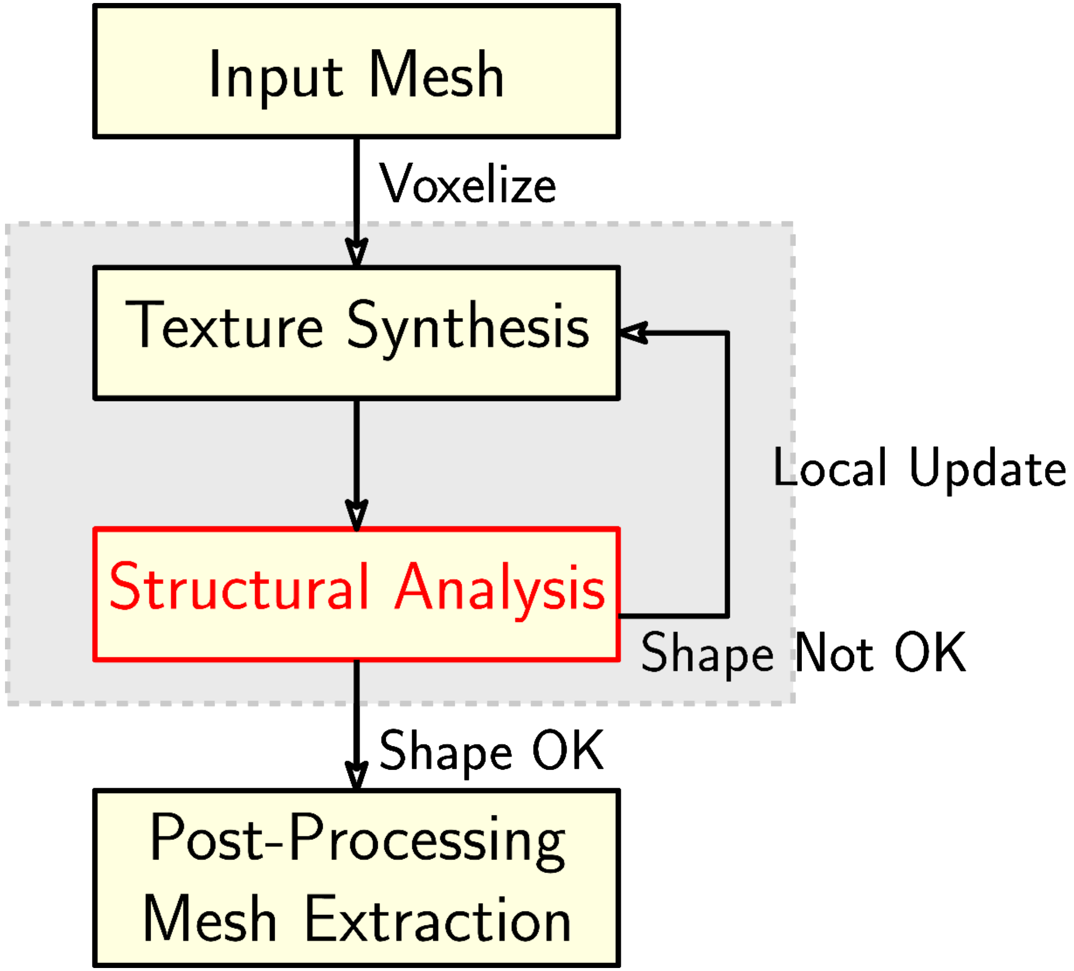
Pipeline


Input Mesh
Voxelize
Texture Synthesis
Local Update
Structural Analysis
Shape Not OK
Shape OK
Post-Processing
Mesh Extraction

Pipeline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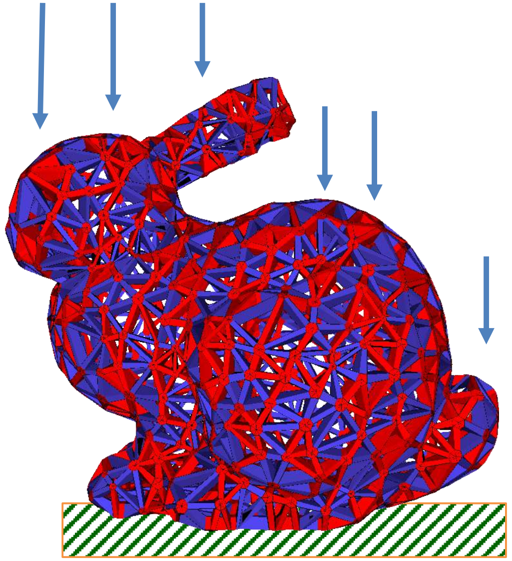
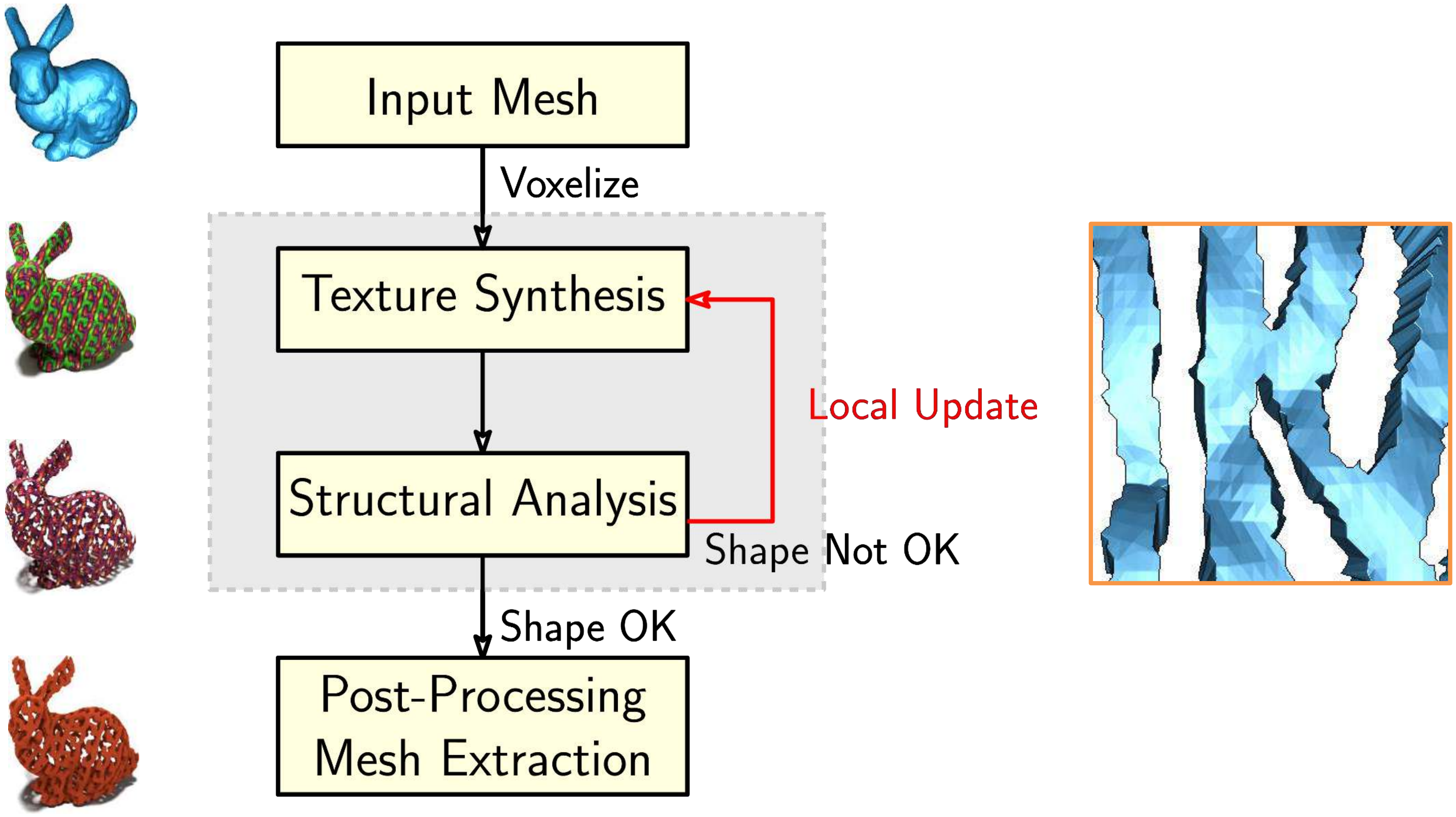


Pipeline


Input Mesh
Voxelize
Texture Synthesis
Local Update
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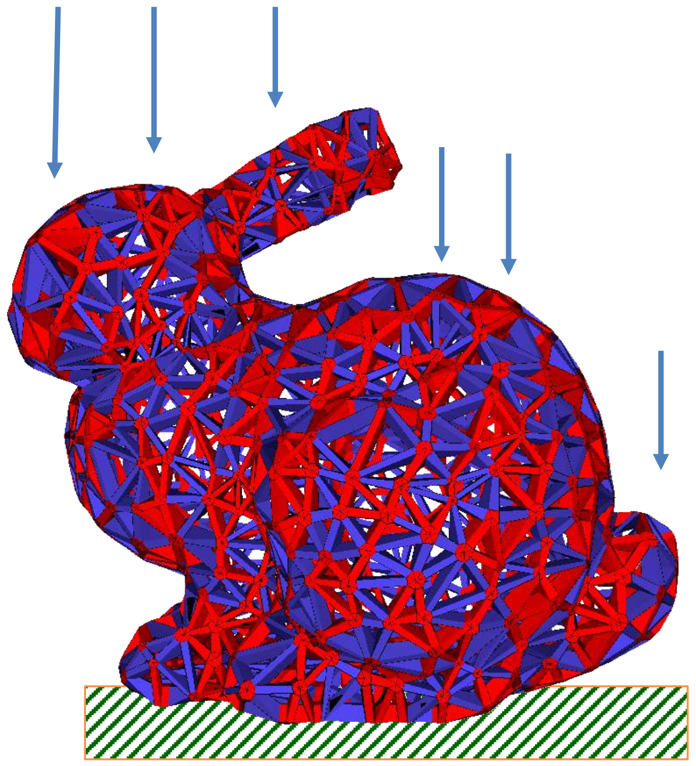
Shape Not OK
Shape OK
Post-Processing
Mesh Extraction

How to evaluate weak parts?

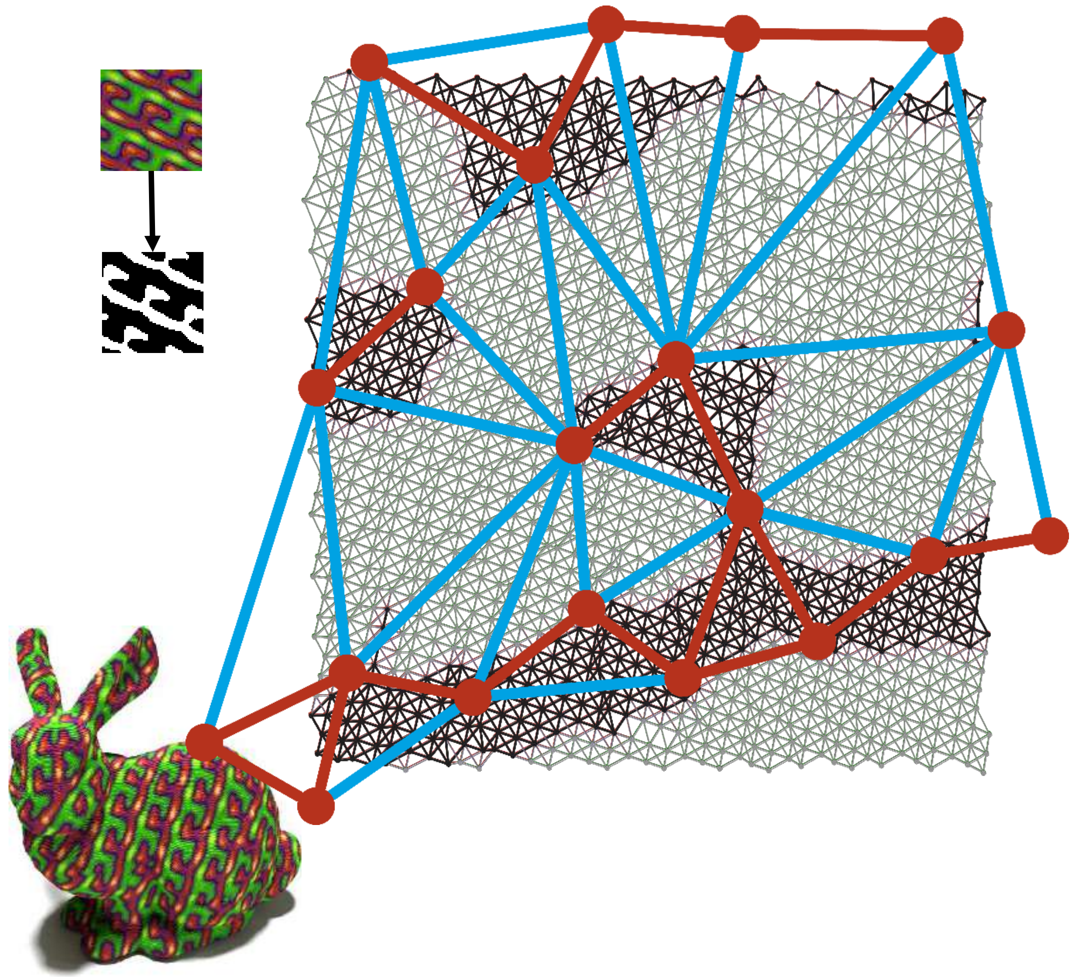
- Similar to SIMP method, we consider ‘weak’ and ‘strong’ material
- Issues:
  - Voxel grid is huge (~ 5M voxels)
  - Weak and strong ➔ hard to converge
  - We need 20-30 iterations synthesis/analysis

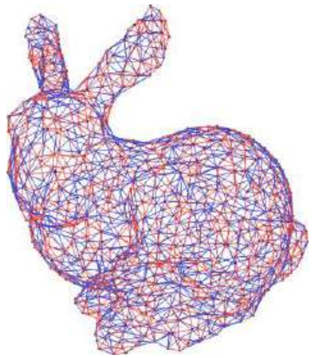
➔ Too expensive

➔ Approximate the pattern



Abstract Pattern Graph

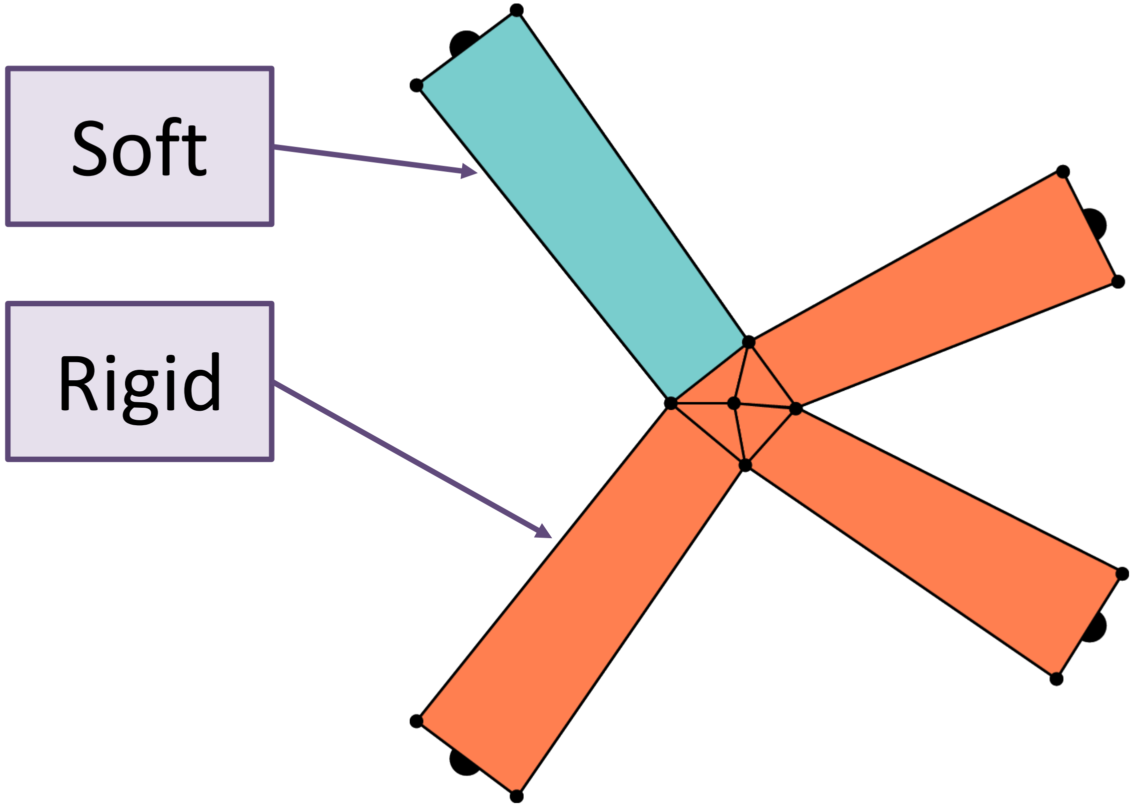




Physical Simulation

- Basic idea: replace graph by finite elements

In 2D: Quad & Triangle
In 3D: Hex & Wedge



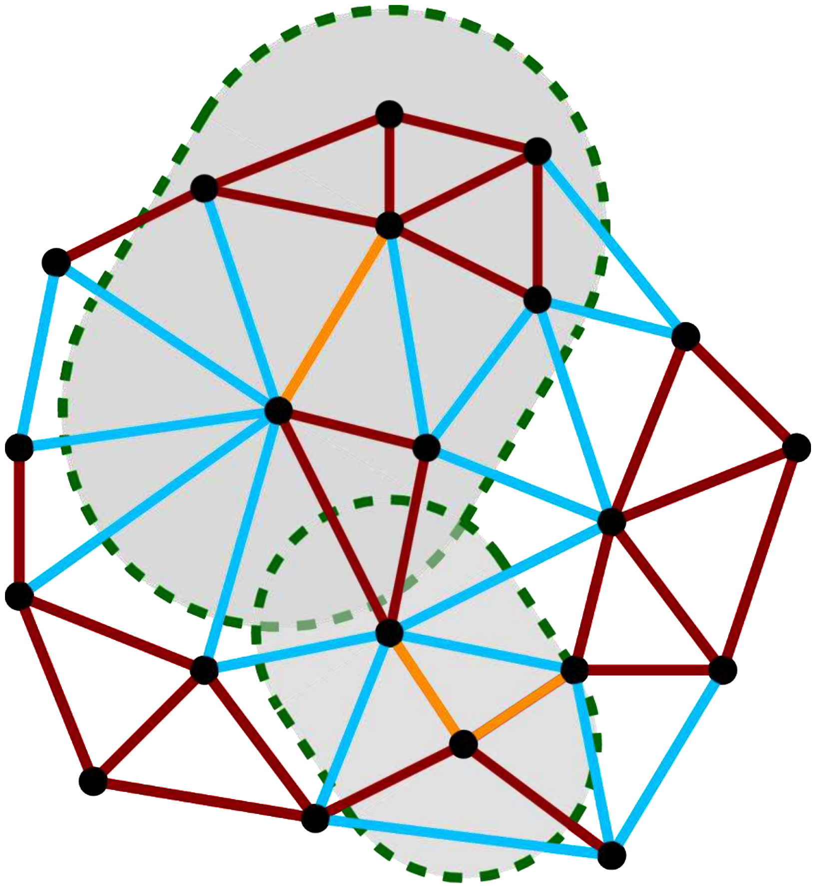
➔ Local planarity assumption
➔ Few elements: fast solution (1s)

Edge Selection Process

Solid

Empty

Selected

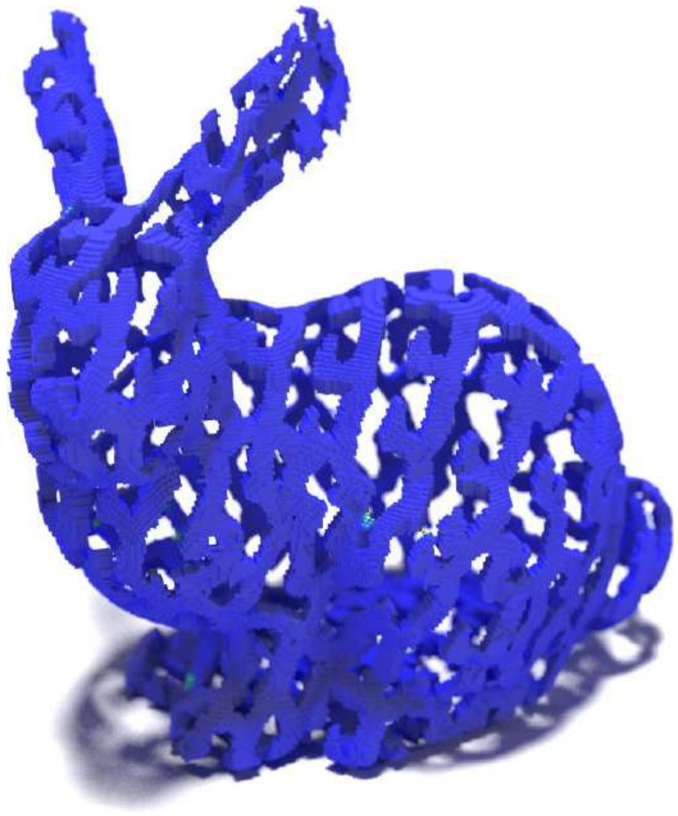


Simulation on the Final Mesh

Stress 99th%

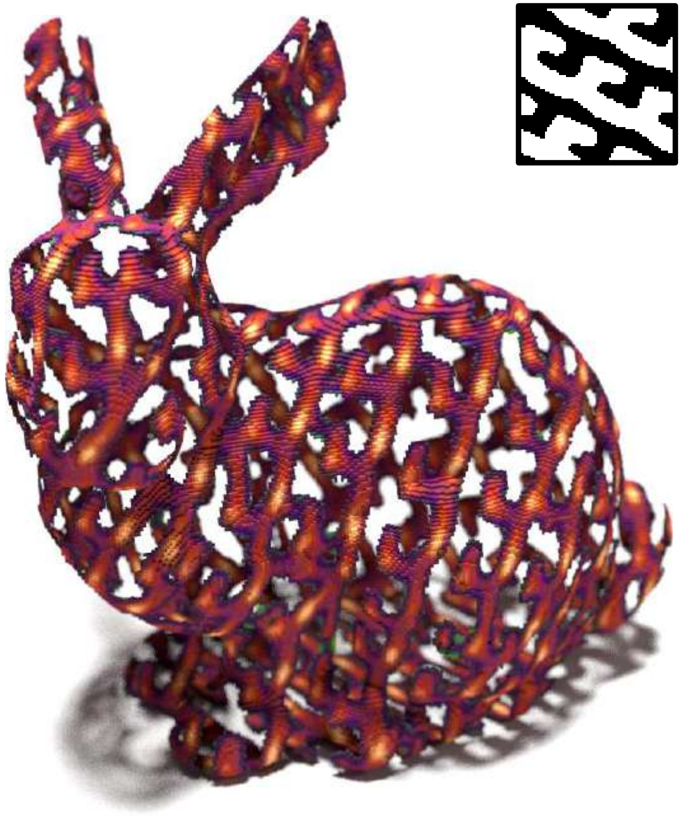
153.9 KPa

30.5 KPa

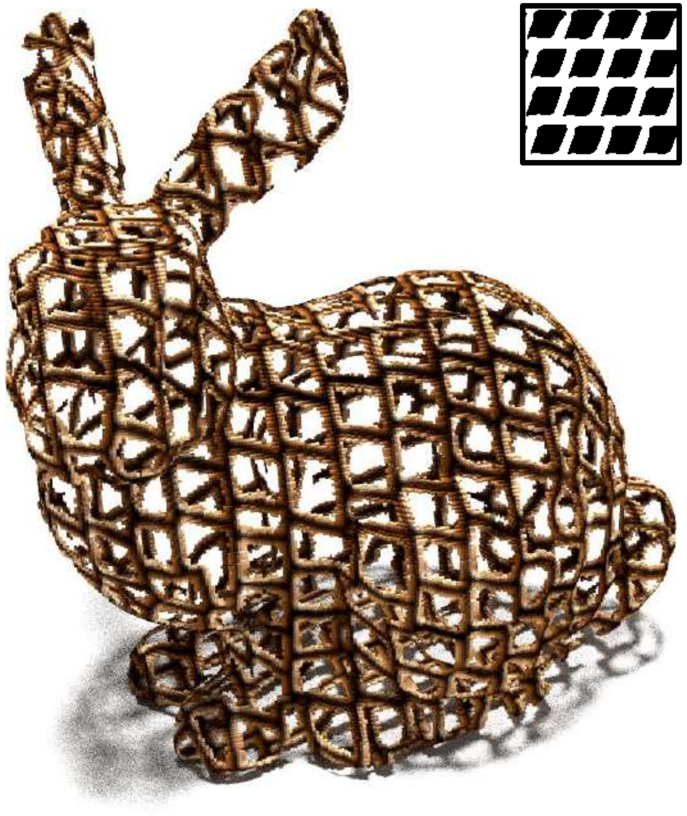


Results – Structure + Color

t_{total}:
34.8s

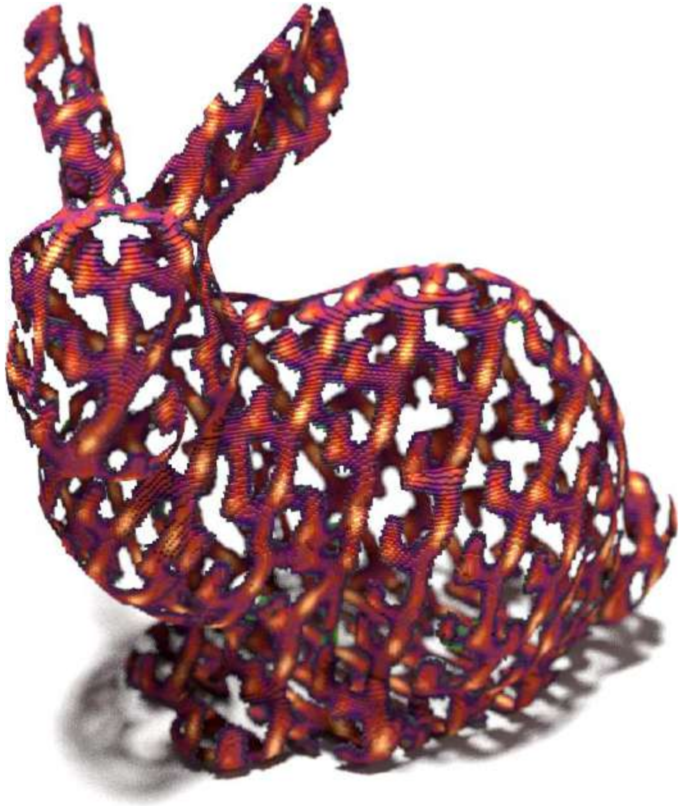
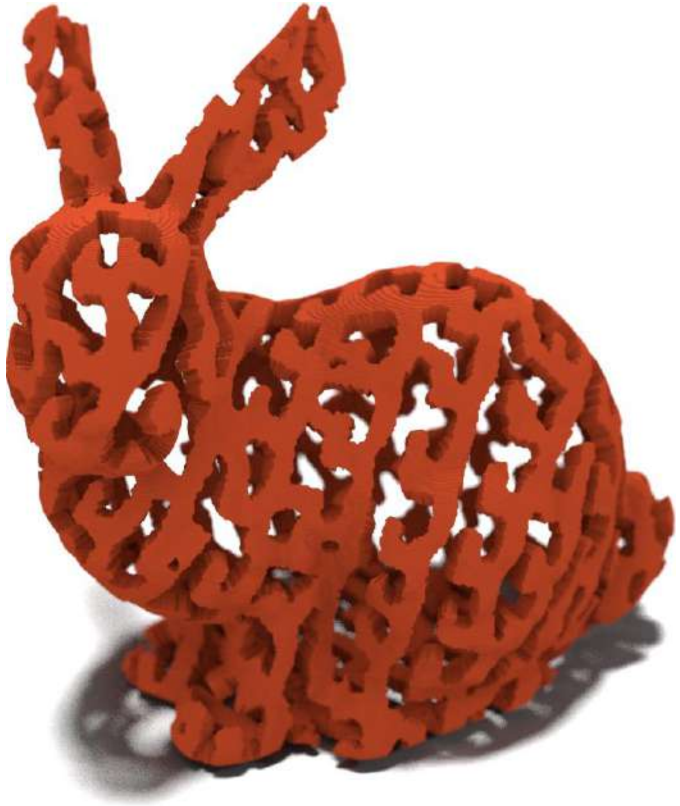
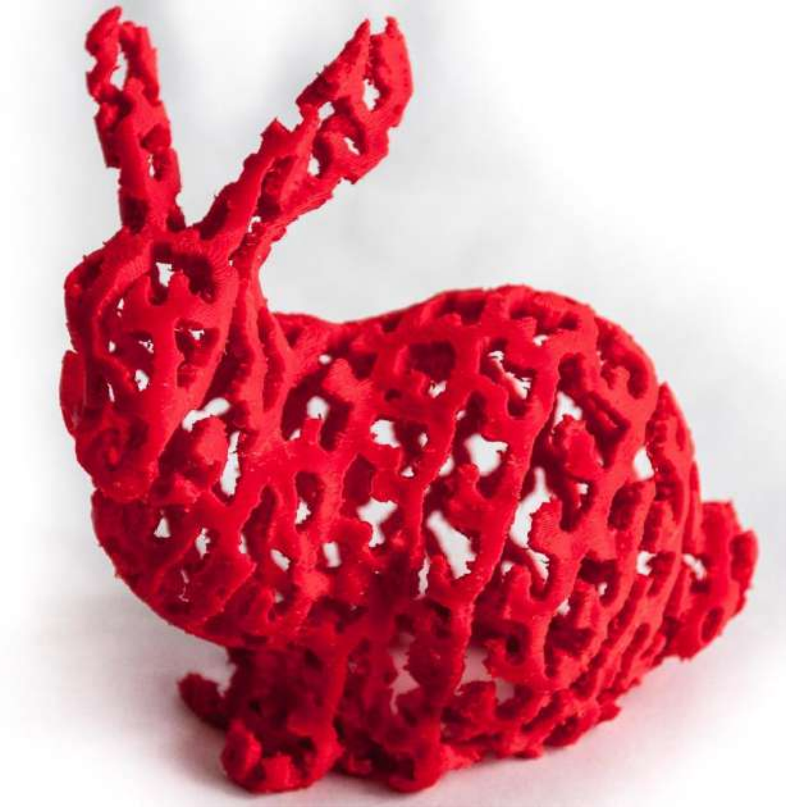


t_{total}:
40.0s



t_{total}:
14.6s

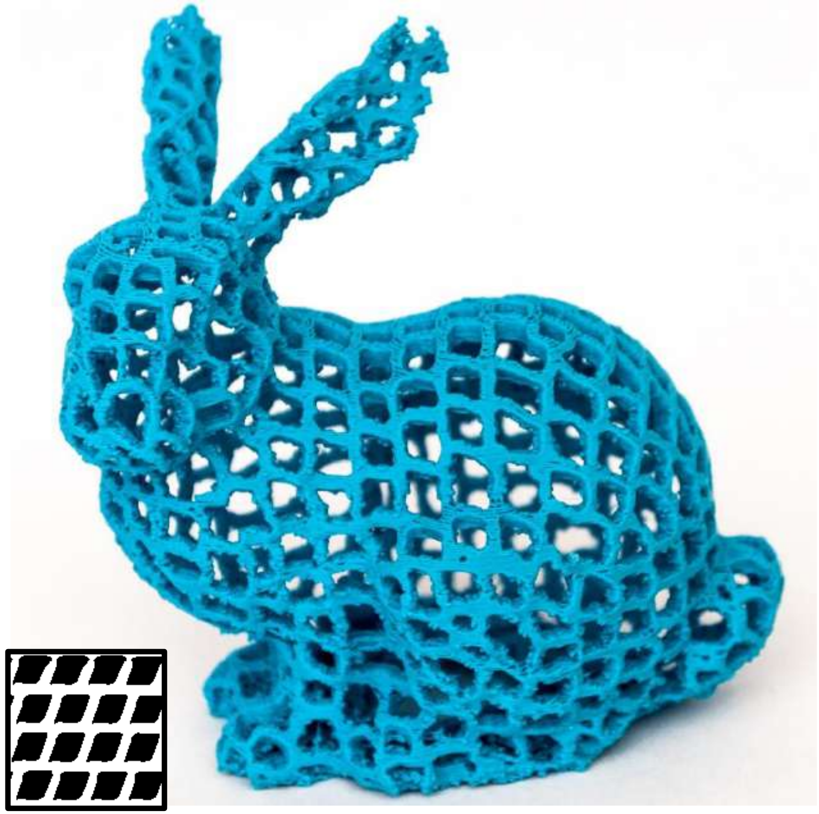
From surface structure to final mesh

Results - Printouts

t_{total}:
52.4s

t_{total}:
11.4s



t_{total}:
14.5s

Other recent references

- **Designing Structurally-Sound Ornamental Curve Networks**
  J. Zehnder, S. Coros, B. Thomaszewski, SIGGRAPH 2016

- **Stenciling: Designing Structurally-Sound Surfaces with Decorative Patterns**
  C. Schumacher, B. Thomaszewski, M. Gross, SGP 2016

- **Synthesis of Filigrees for Digital Fabrication**
  W. Chen, X. Zhang, S. Xin, Y. Xia ,S. Lefebvre and W. Wang, SIGGRAPH 2016

All these works use a different point of view: discrete element distributions

Key ideas for structure synthesis

Pattern is stochastic

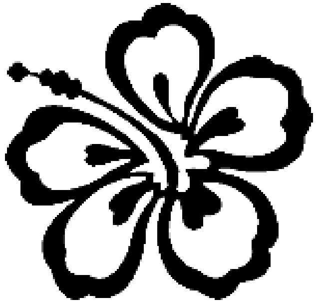
- Exhibits degrees of freedom
- Use pattern itself to locally reinforce structure

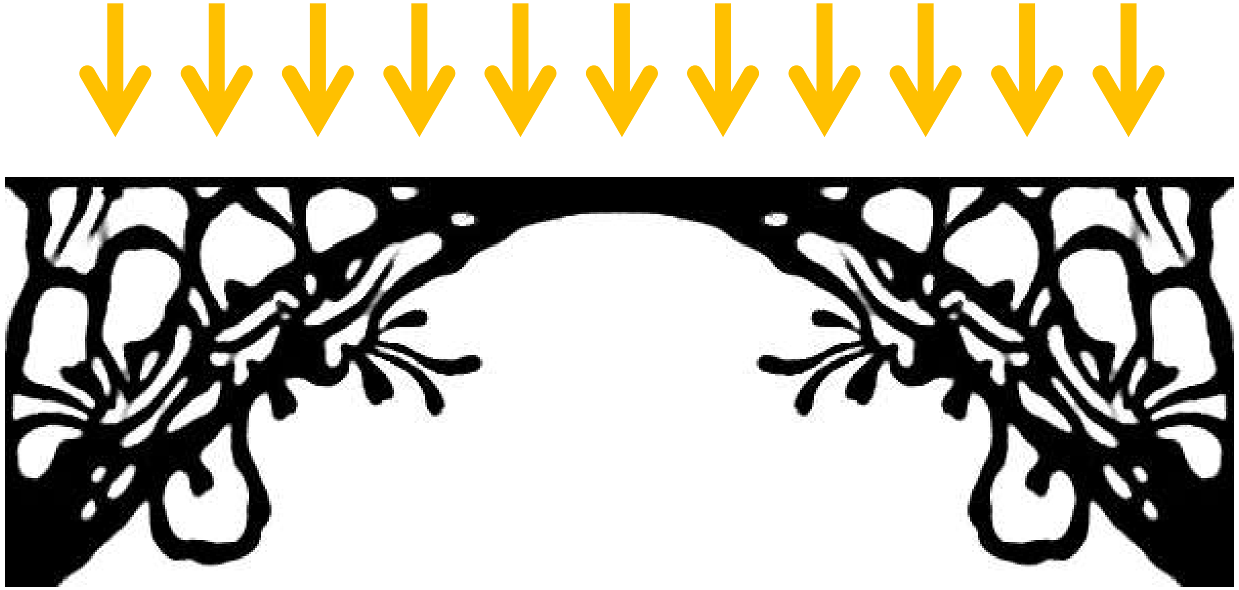
Exemplar specifies local geometry

- Large scale arrangement can be optimized ‘orthogonally’
- Combination with topology optimization?

Our Goal

Exemplar





Synthesize shapes under structural and appearance objectives

Local geometry

minimise

$$E(\Omega) = \boxed{\int_{p \in \partial\Omega} \min_{q \in \partial \mathrm{X}} D(N_{\mathrm{X}}(q), N_{\Omega}(p))}$$

Local geometry

$N_{\mathrm{X}}(q)$

$N_{\Omega}(p)$

X Example shape

Ω Synthesized shape

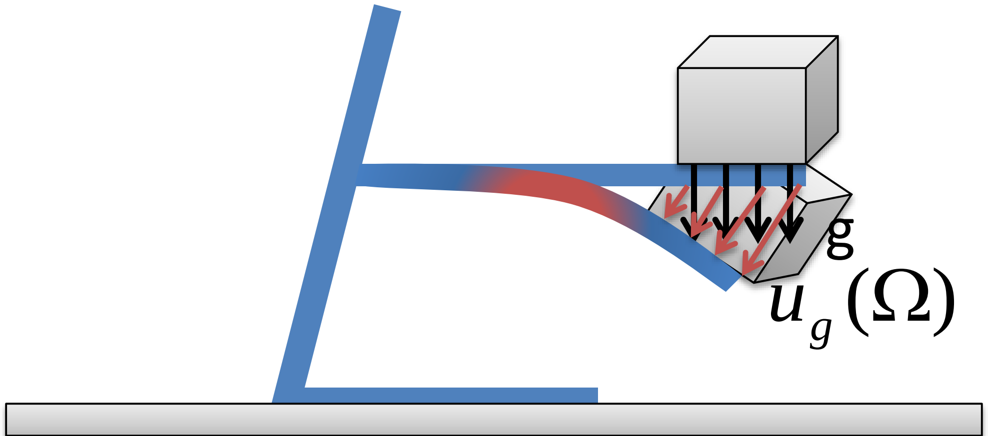
Structural properties

minimise

$$E(\Omega) = \boxed{\int_{\partial\Omega} g.u_g(\Omega)dx}$$

compliance

rigidity



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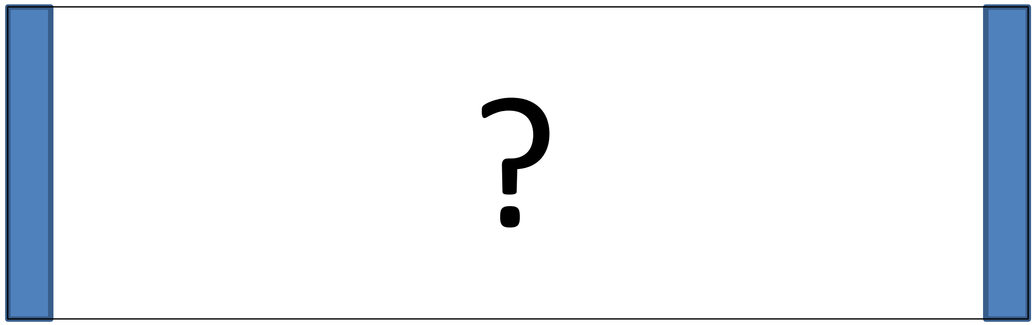


Gravity

Topology optimization [Osher, Allaire, Sigmund]

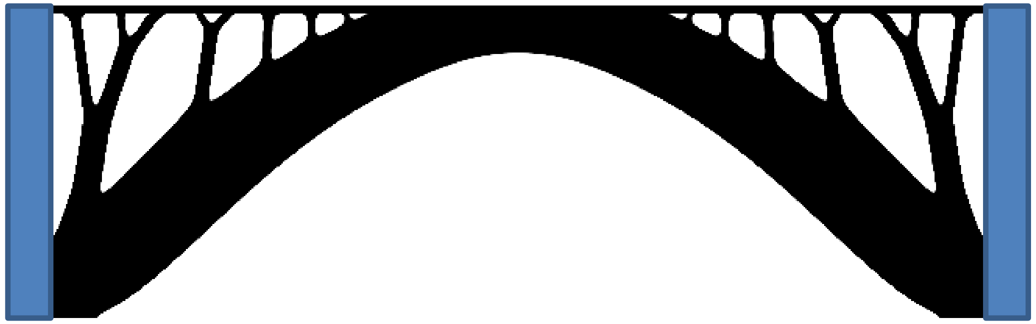
Structural properties

minimise

$$E(\Omega) = \boxed{\int_{\partial\Omega} g.u_g(\Omega)dx}$$

compliance

rigidity



Gravity

Topology optimization [Osher, Allaire, Sigmund]

Challenge

minimise

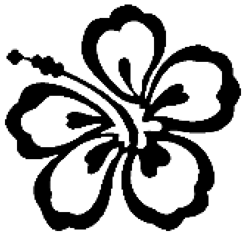
$$E_0(\Omega) = \boxed{\int_{p\in\partial\Omega} \min_{q\in\partial \mathrm{X}} D(N_\Omega(p), N_\mathrm{X}(q))}$$

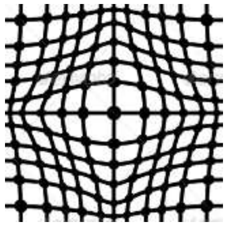
local geometry

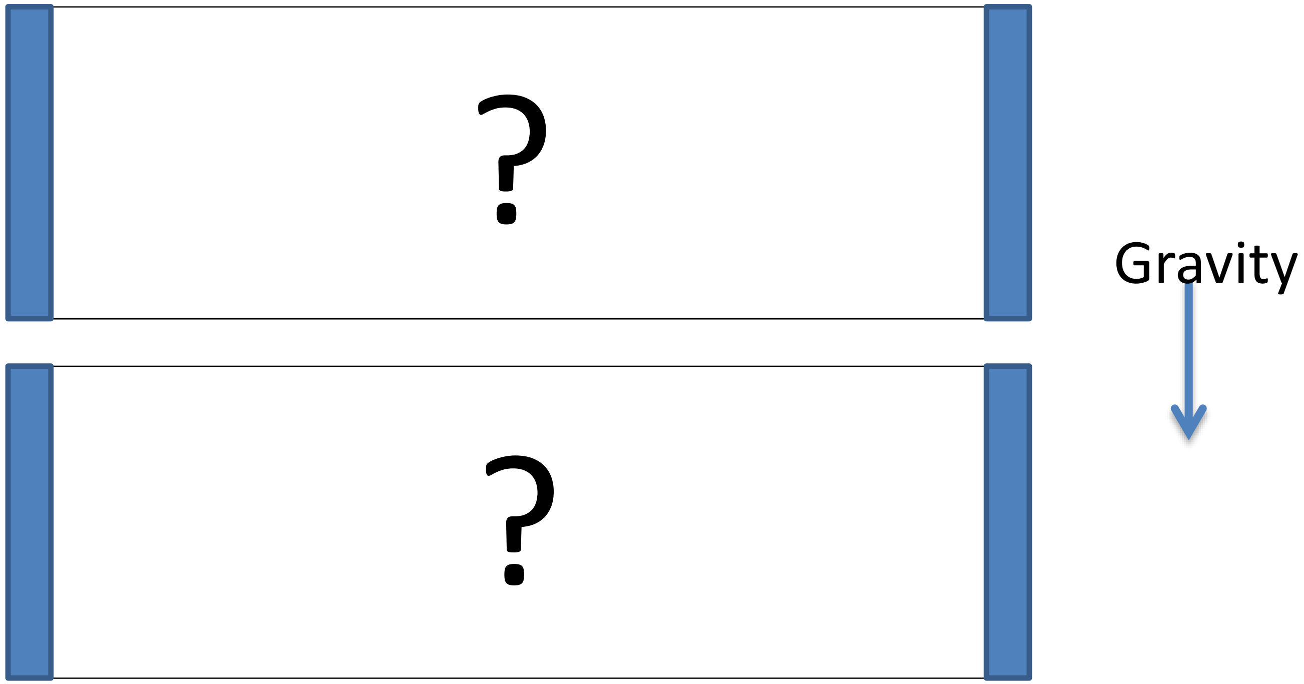
minimise

$$E_1(\Omega) = \boxed{\int_{\partial\Omega} g.u_g(\Omega)dx}$$

rigidity







Challenge

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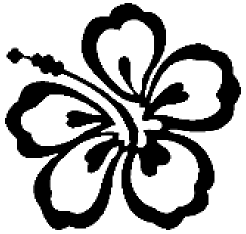
$$E_0(\Omega) = \boxed{\int_{p\in\partial\Omega} \min_{q\in\partial X} D(N_\Omega(p), N_X(q))}$$

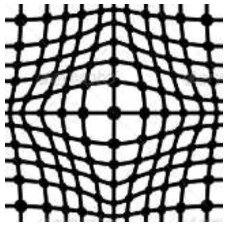
local geometry

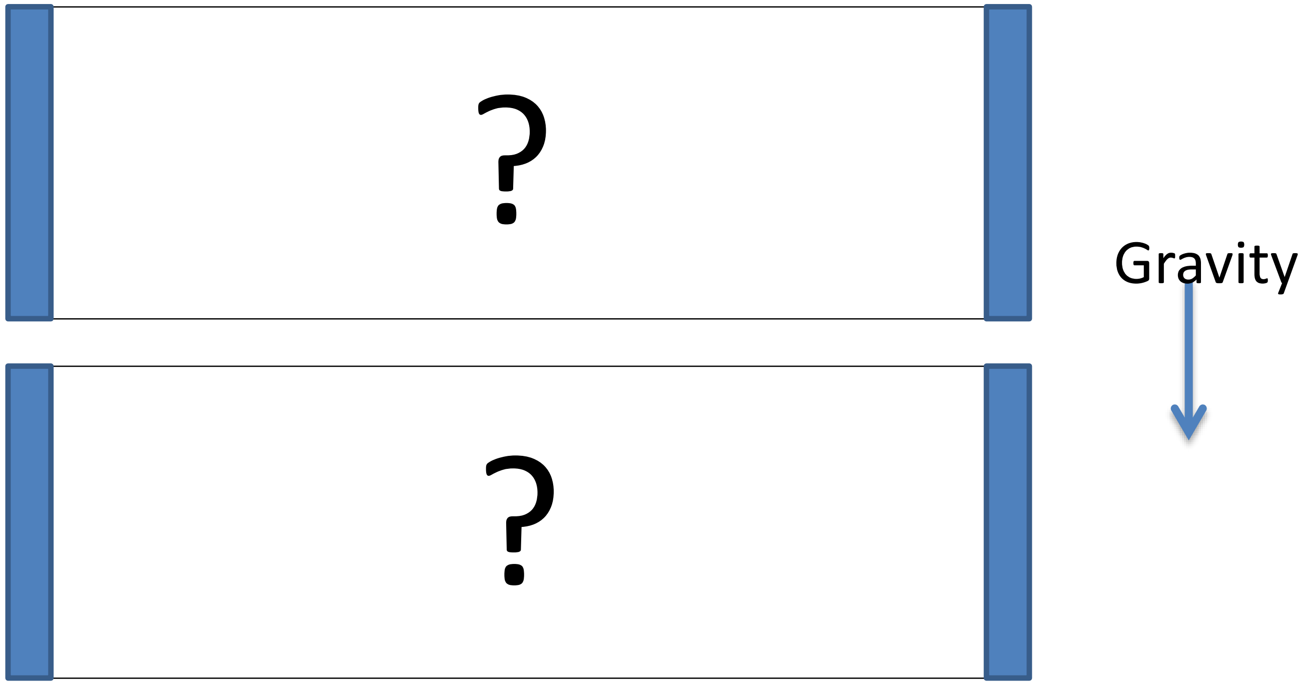
minimise

$$E_1(\Omega) = \boxed{\int_{\partial\Omega} g\,\mu_g(\Omega)dx}$$

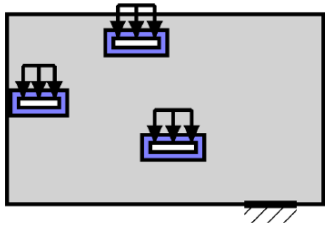
rigidity

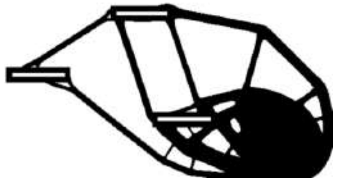




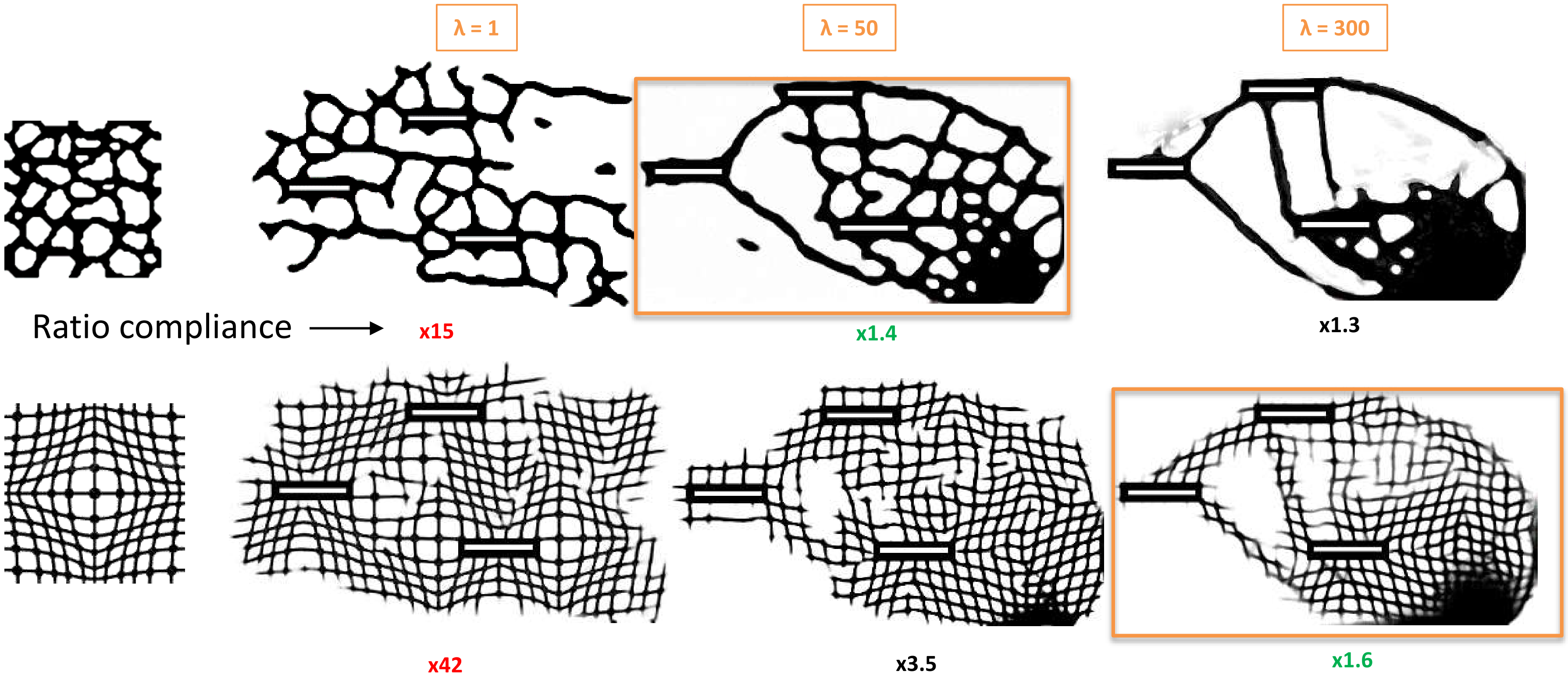


Weighted sum





$\cancel{\text{Minimize } G(x) + \lambda\, C(x)}$



Appearance + rigidity

minimise

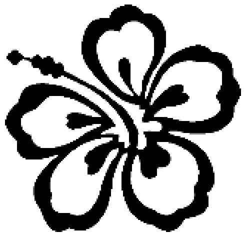
$$E_0(\Omega) = \boxed{\int_{p\in\partial\Omega} \min_{q\in\partial \mathrm{X}} D(N_\Omega(p), N_{\mathrm{X}}(q))}$$

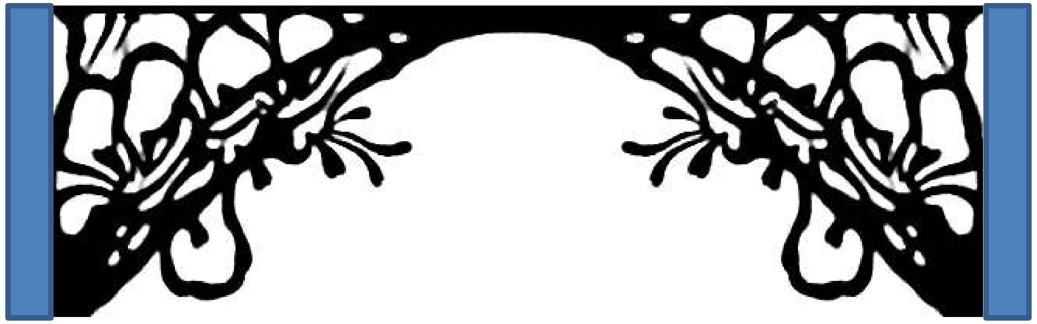
appearance

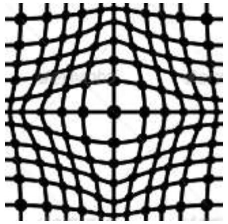
such that

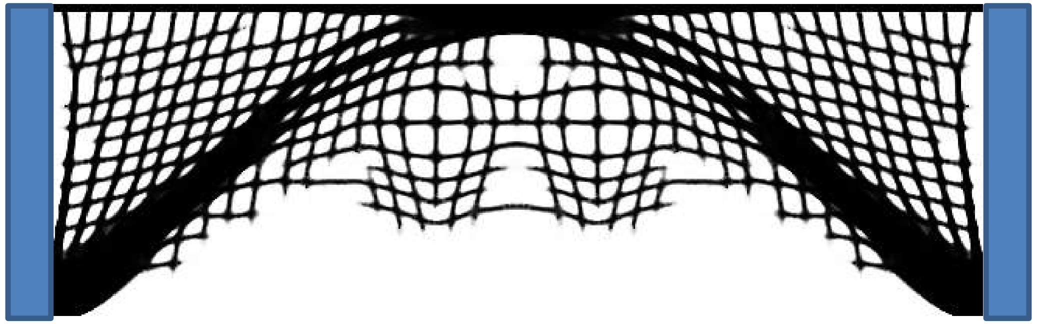
$$E_1(\Omega) = \boxed{\left(\int_{\partial\Omega} g.u_g(\Omega)dx\right) < \alpha.\boxed{C_{\max}}}$$

rigidity









Gravity

Solver

Not great due to combinatorial matching

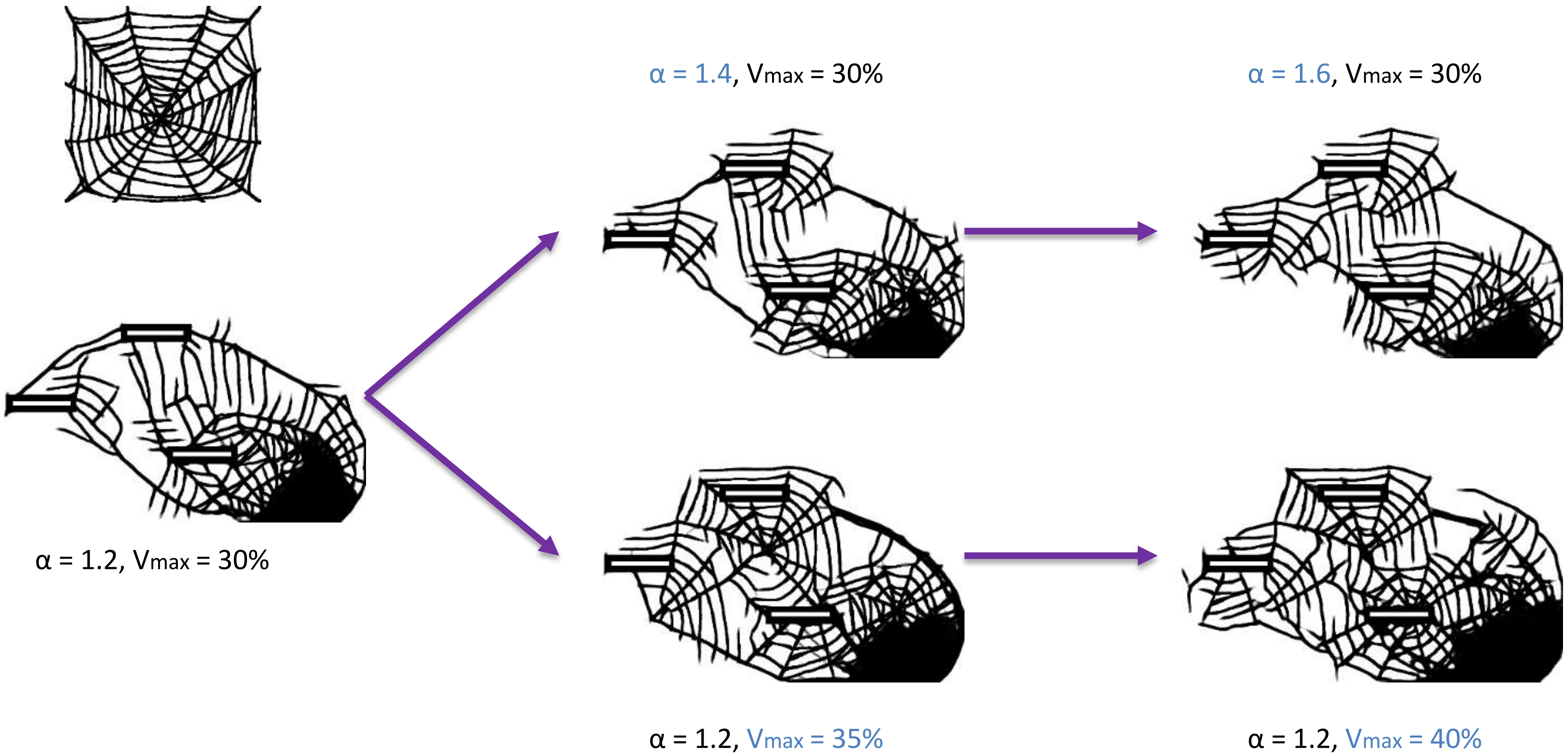
Appearance objective
- Neighborhood matching [Barnes09, Busto10, Kaspar15]
- Derivatives **A(x)**

Compliance constraint
- Linear elasticity (FEM)
- Derivatives **C(x)**

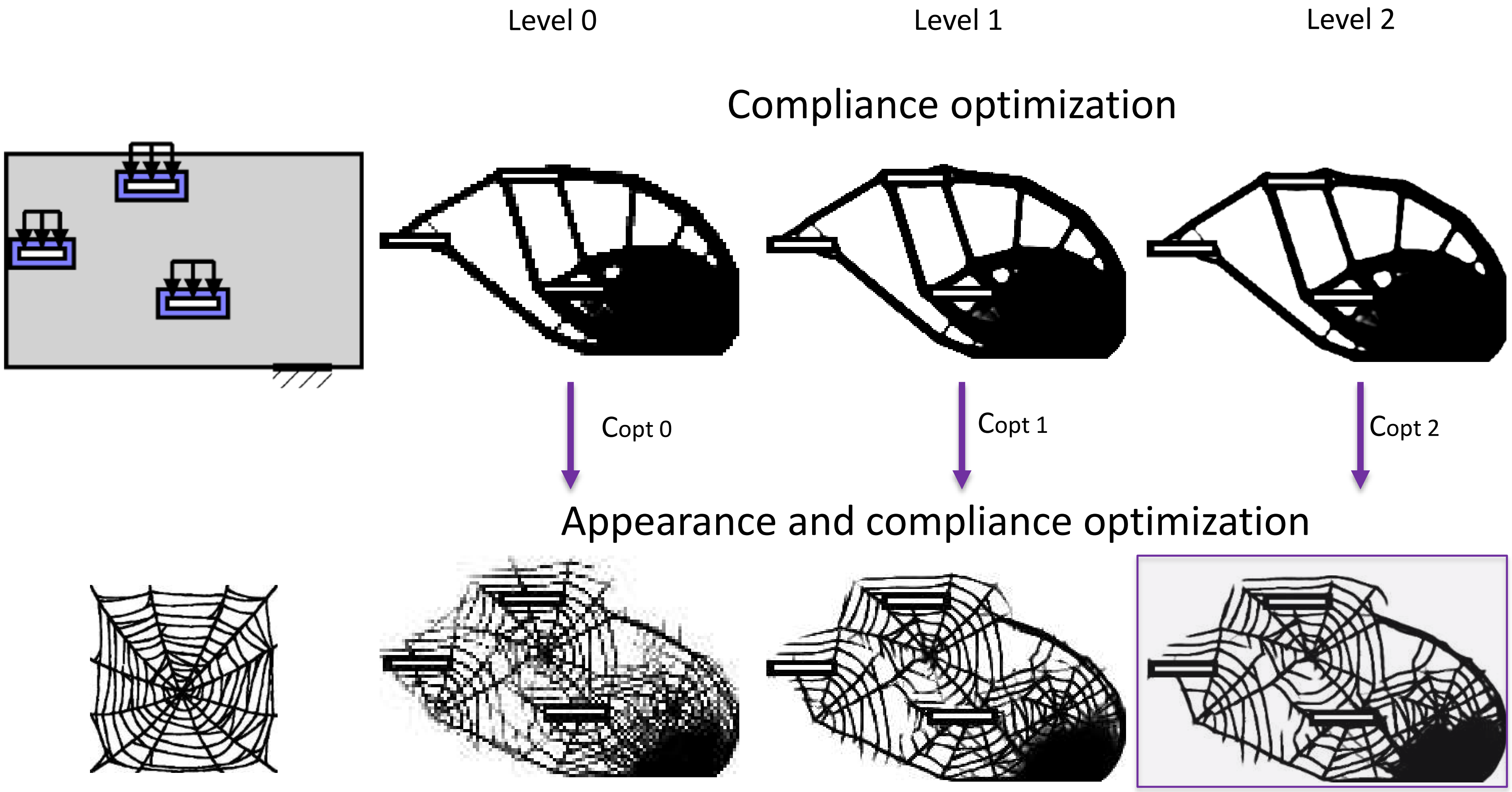
Volume constraint
- Derivatives **sum(x)**

Gradient-based Optimization
GCMMA [Svanberg95]

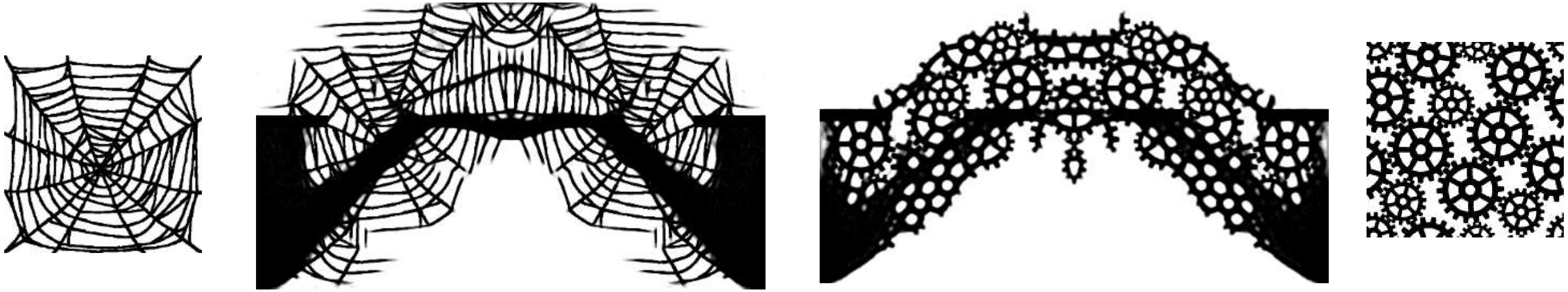
Compliance Relaxation



Multiresolution

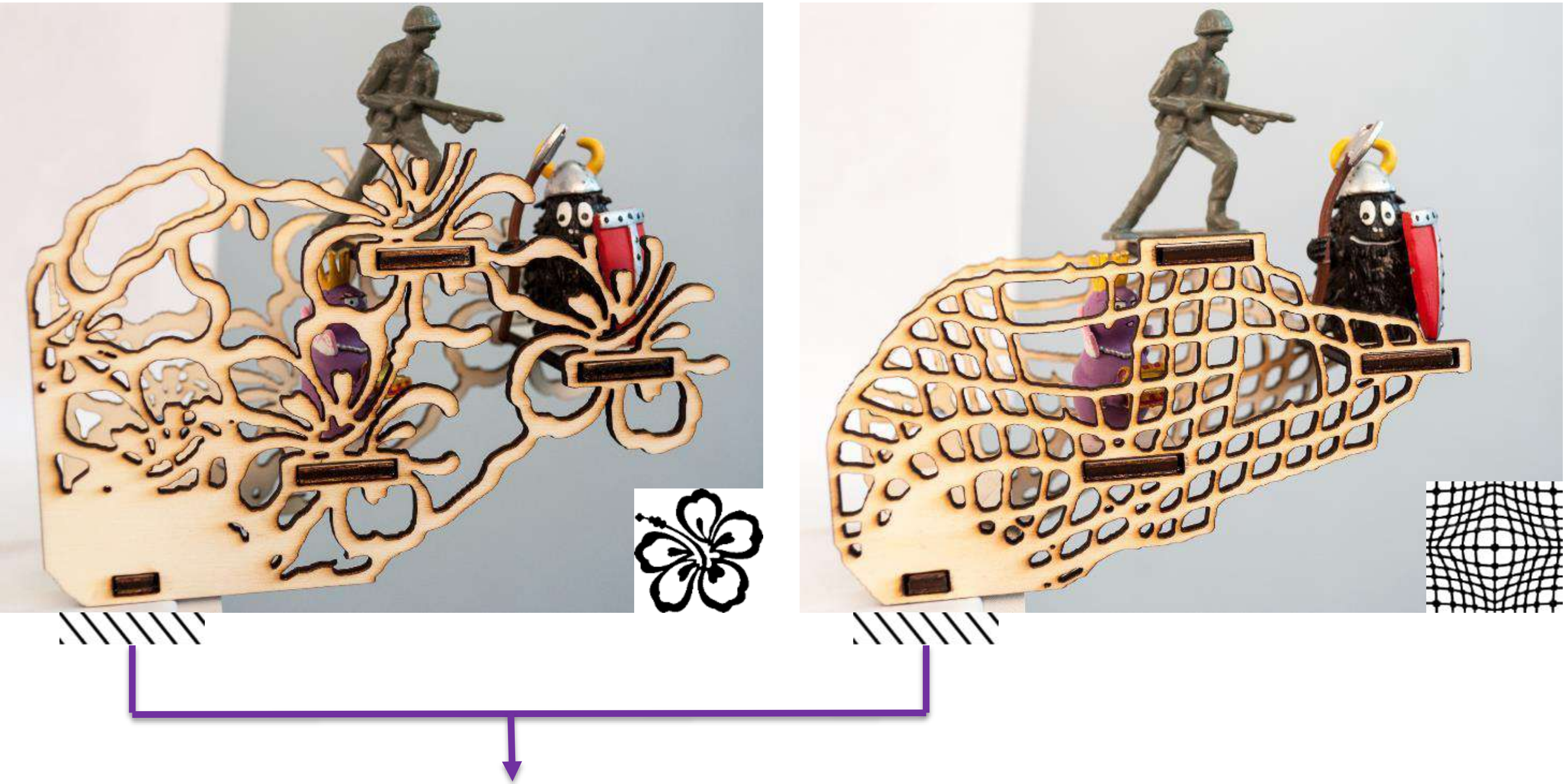


Fabricated Objects



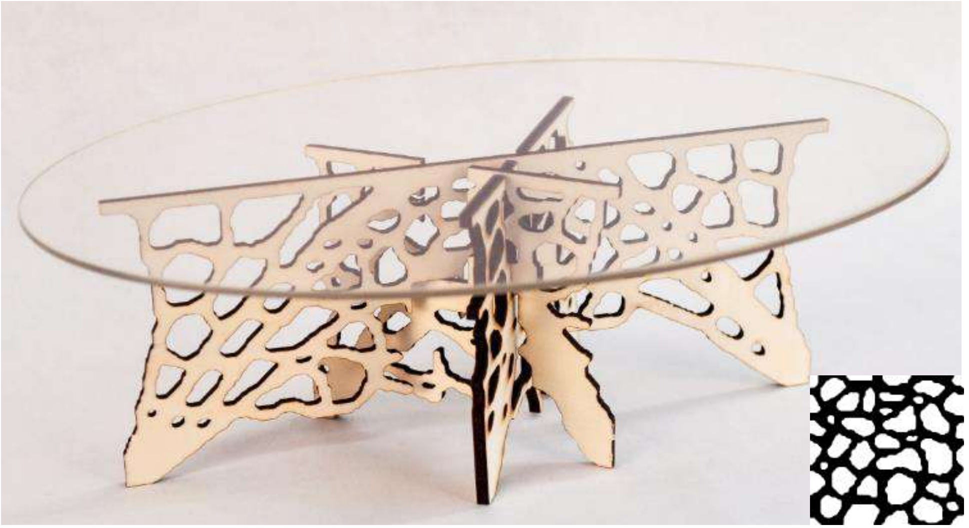
Contour extraction

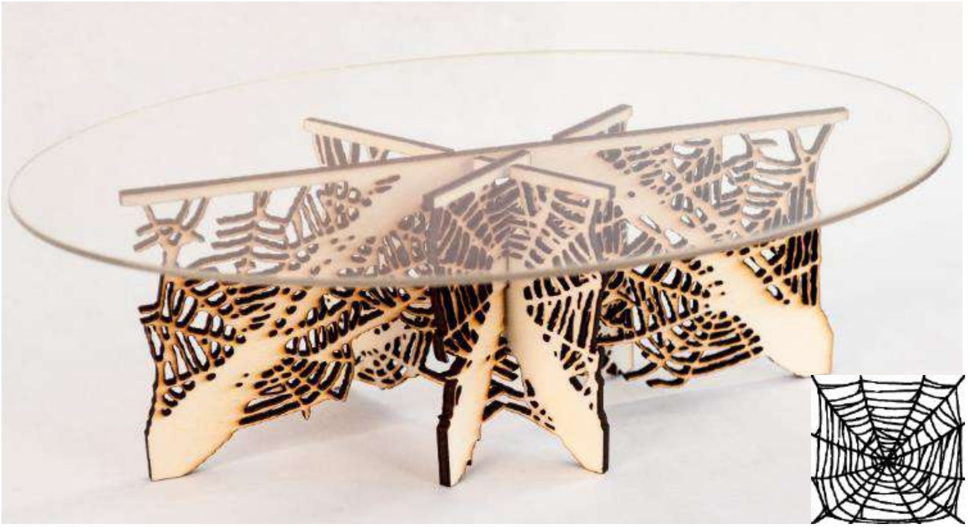
Fabricated Objects: Shelves



Floor attachment

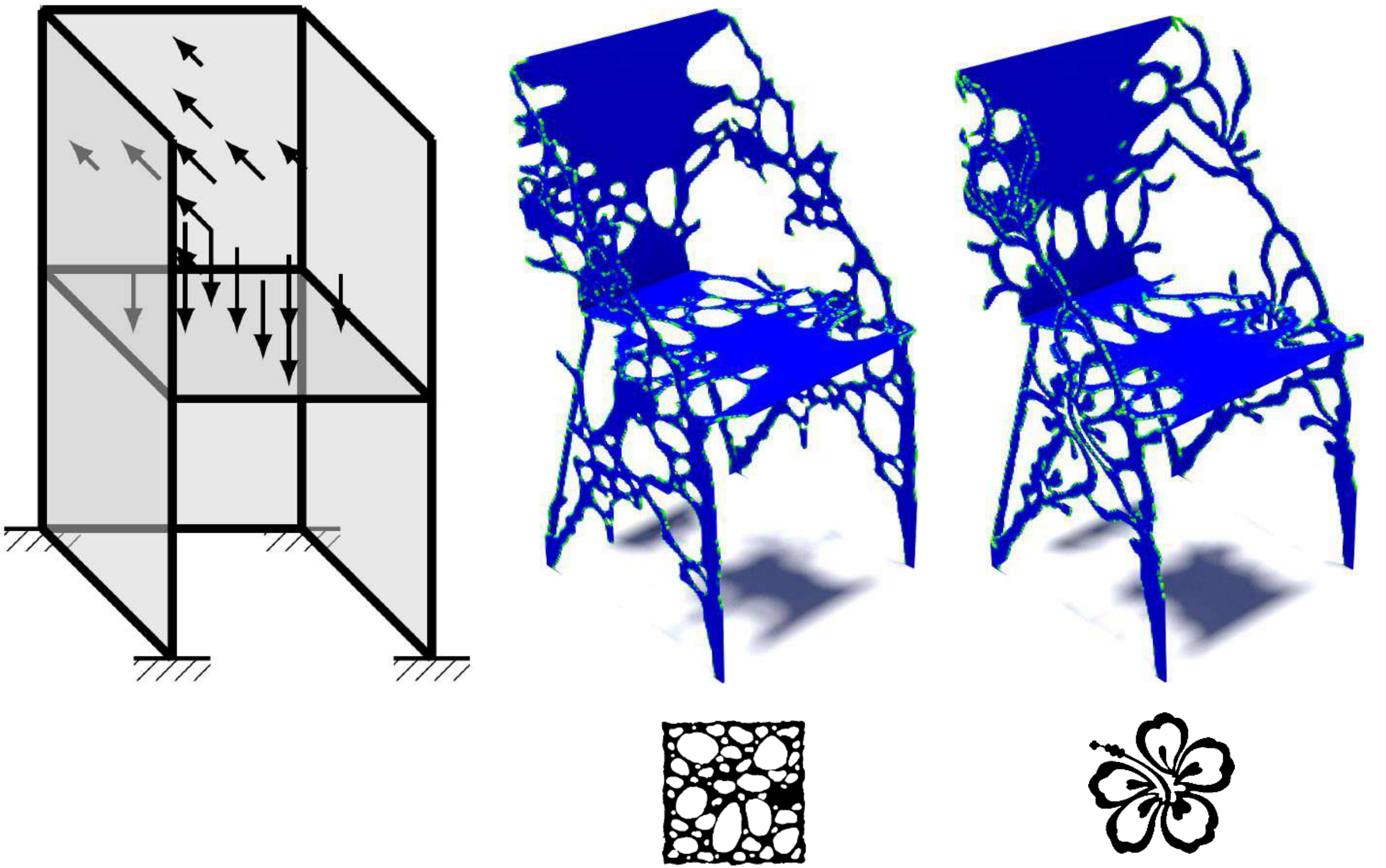
Fabricated Objects: Tables



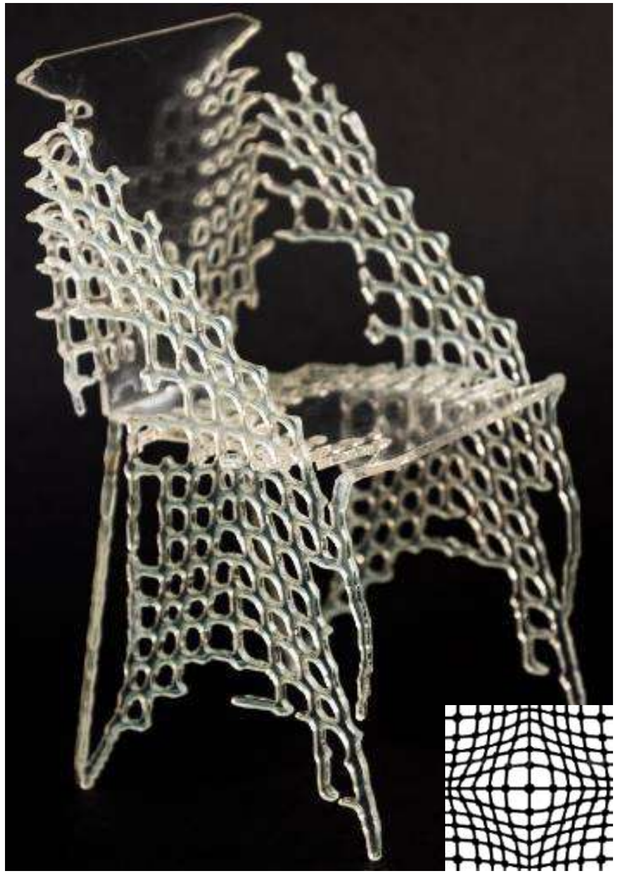


Fabricated Objects: Phone Stands

3D Structures



Fabricated Objects: Chairs



Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g. erosion)

We will see both in the context of fabrication

Texture Synthesis

- Three main directions
  - By-example synthesis
  - Procedural synthesis
  - Simulation (e.g. erosion)

We will see both in the context of fabrication

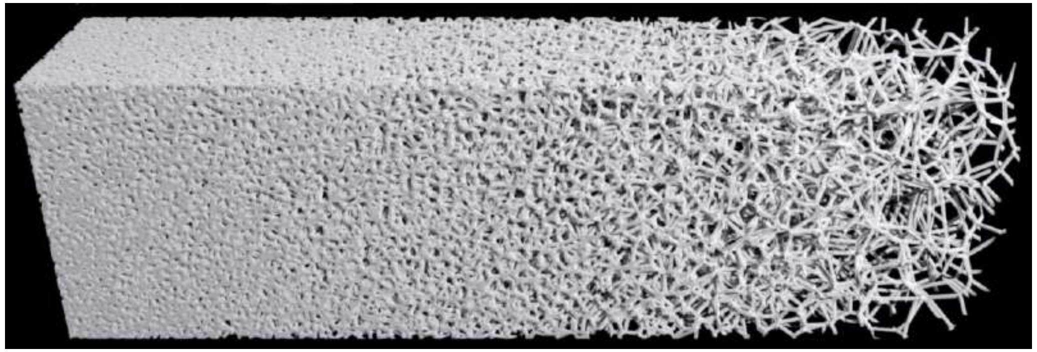
Foams in nature

Coral reef

Metallic foam (chemical reaction)

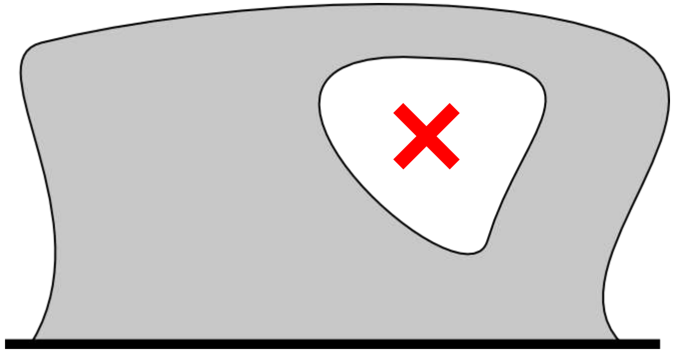
Challenges: scale, fabricability, mechanical properties

- Data size

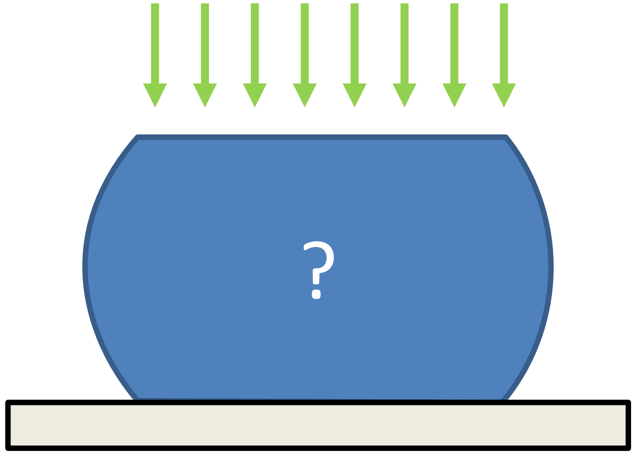


4 GB (.ply)

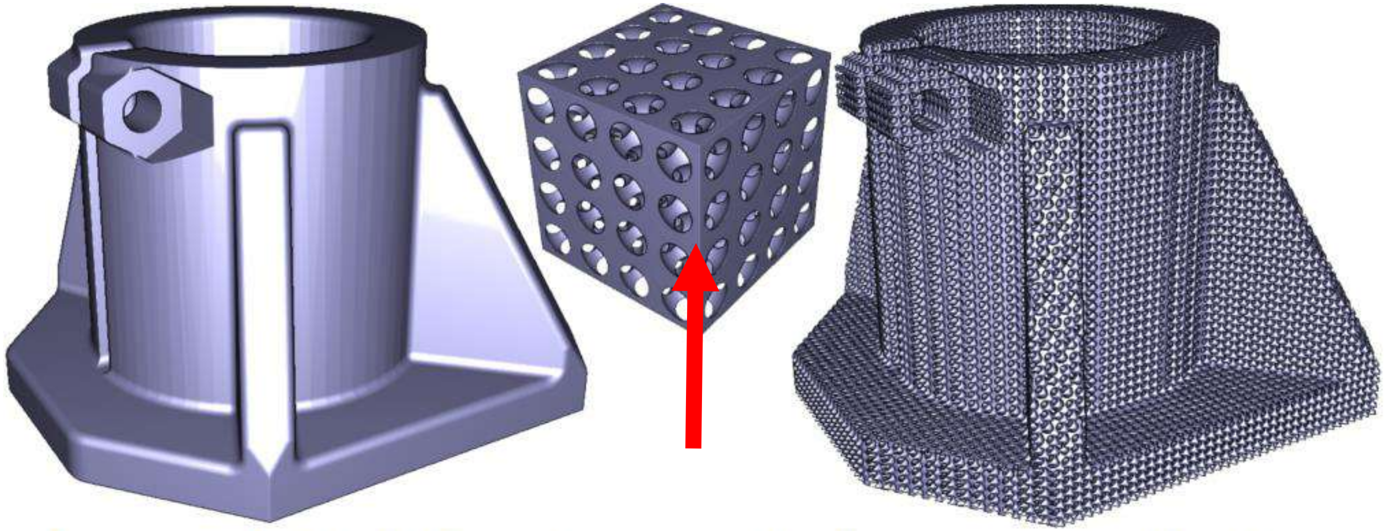
- Fabrication



- Mechanical properties

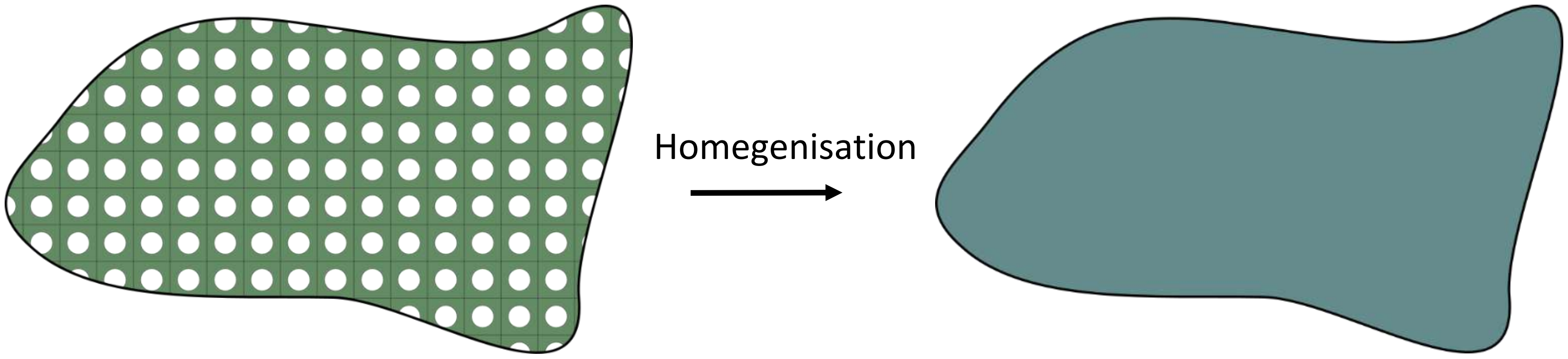


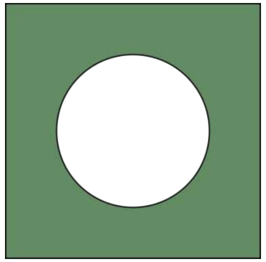
Standard approach: periodic structures



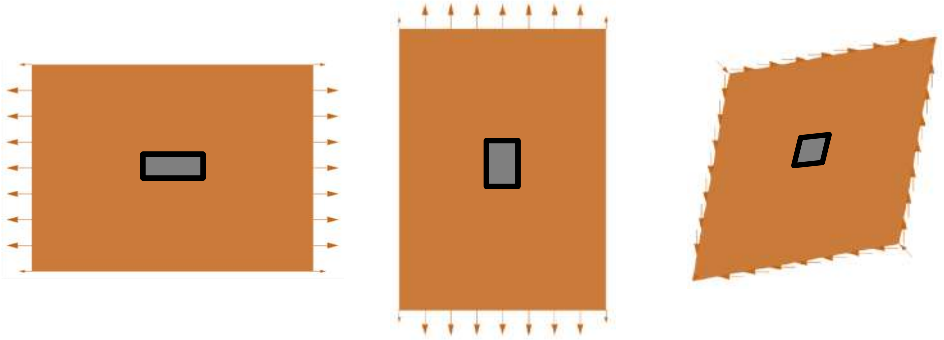
```
spheres = implicit(bx:min_corner(), bx:max_corner(), [[
float distanceEstimator(vec3 p)
{
  const float diameter = 1.0;
  p = p / diameter;
  float d = length(p-ivec3(floor(p))-vec3(0.5,0.5,0.5))-0.6;
  return -d*diameter;
}
]])
```

Homogenisation



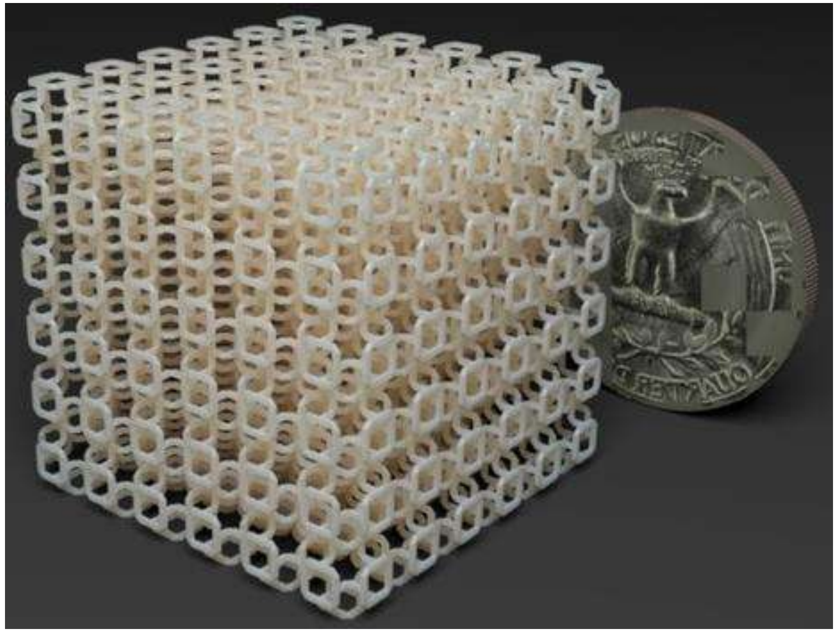


Representative
Volume Element (**RVE**)

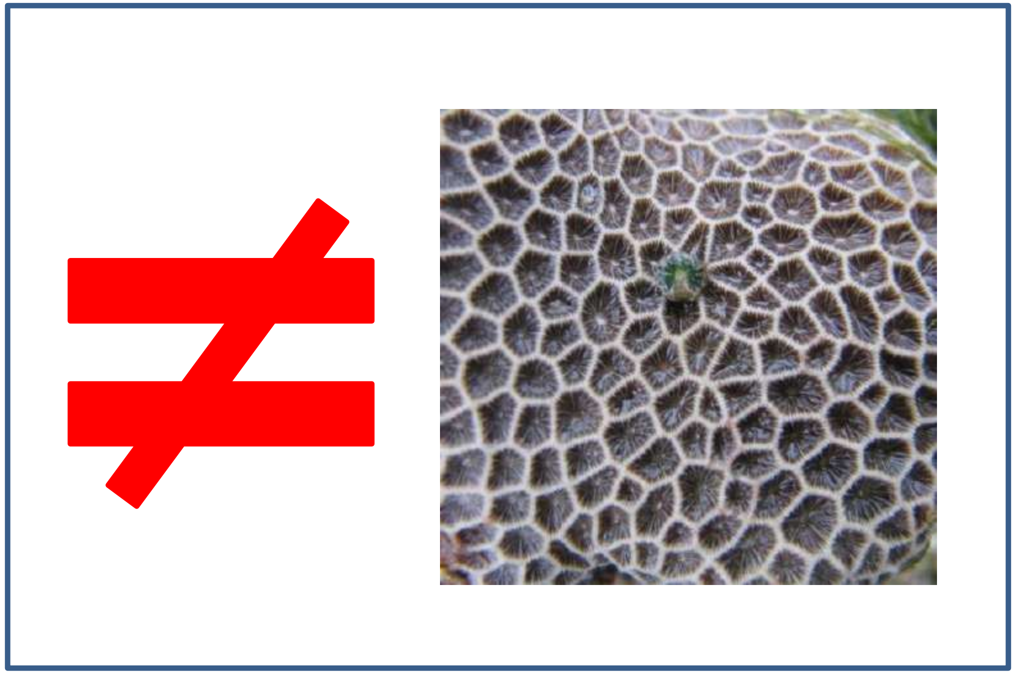


Homogenized elasticity tensor
[Andreassen and Andreasen 2014]

Drawbacks

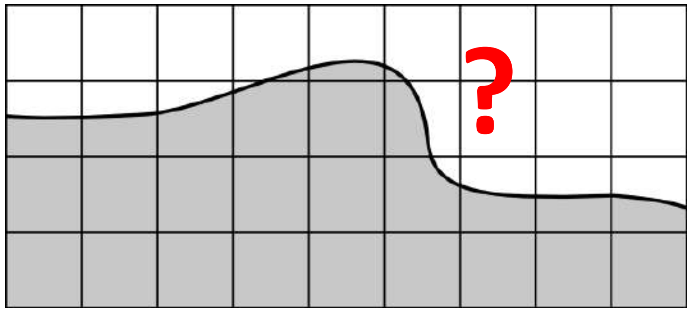


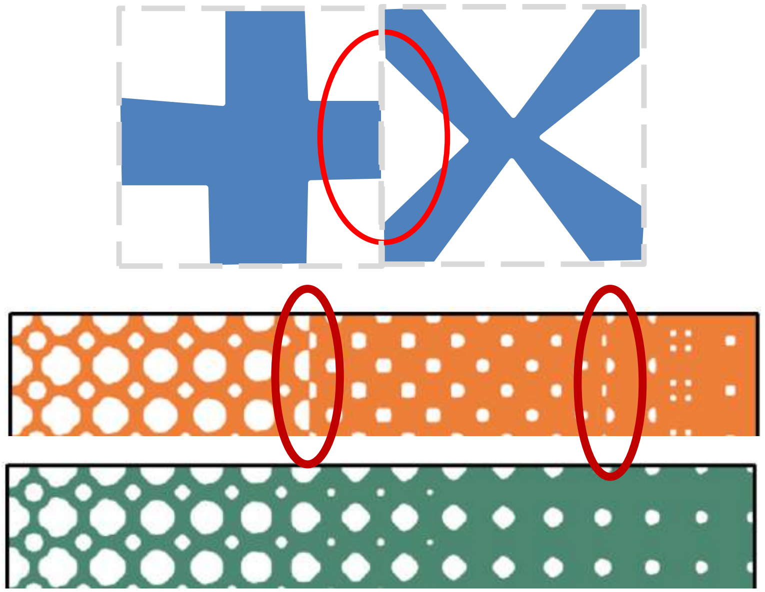
[Pannetta et al. SIGGRAPH 2015]



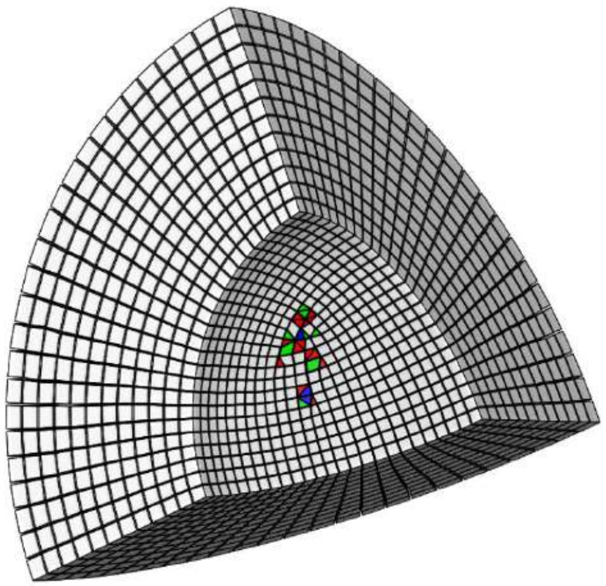
Periodic grid

- Mapping?
  - Hard problem
- Graded properties:
  - Possible, but transitions?





[Schumacher et al. SIGGRAPH 2015]



Hexahedral-dominant meshing
[Sokolov et al. 2015]

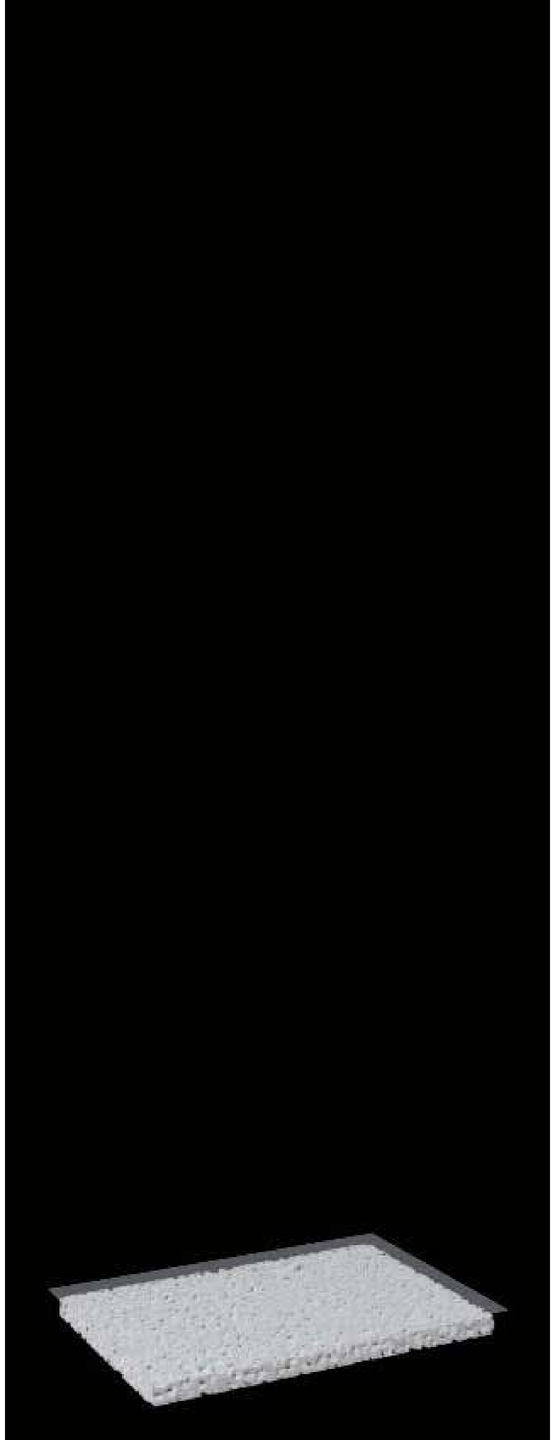
Procedural Voronoi Foams

- **Aperiodic, stochastic, stationary**
  Mimics nature.

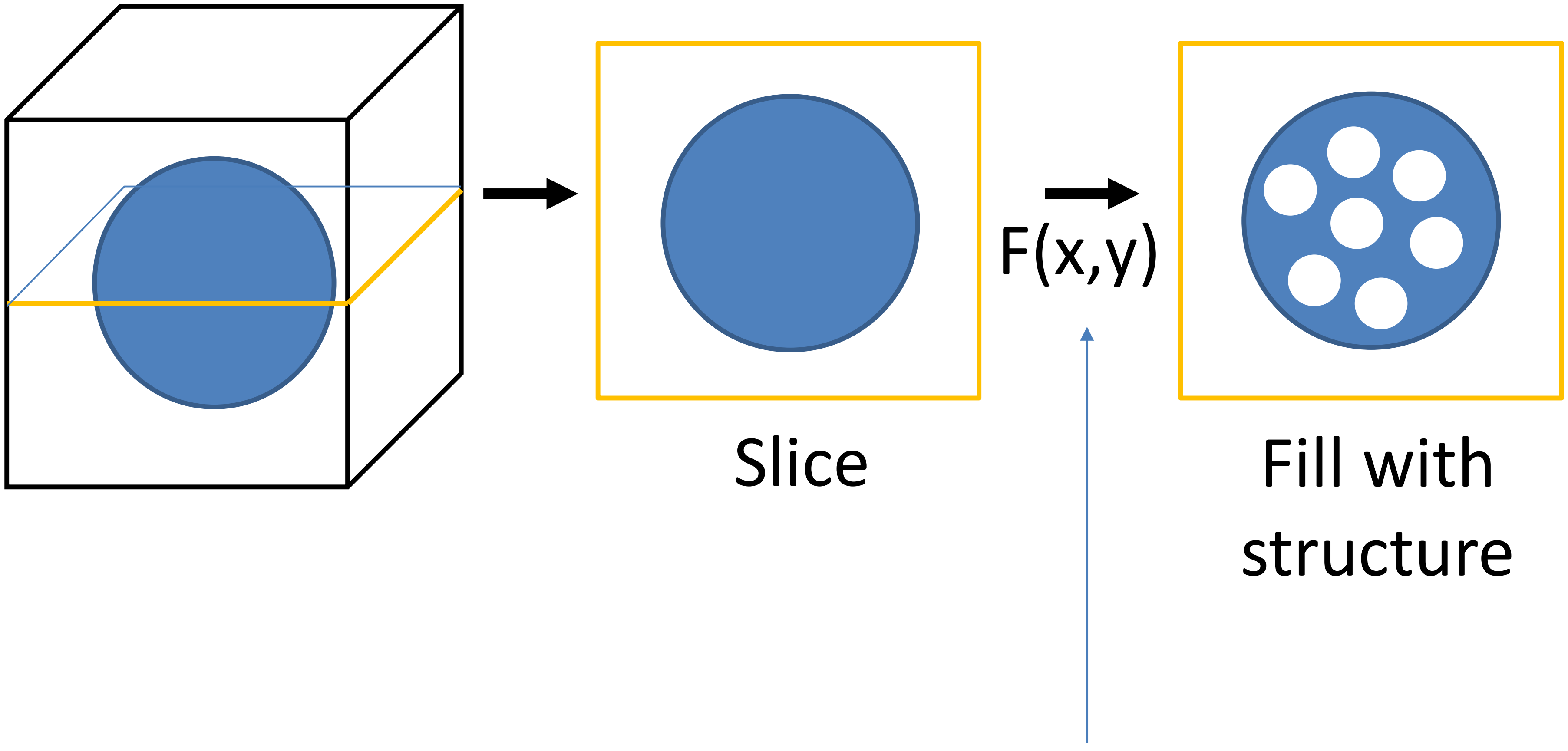
- **Trivially scales**.
  O(1) time + memory.

- **Fabricable**.
  Few pockets, connected, thickness ok.

- ***Controllable elasticity***



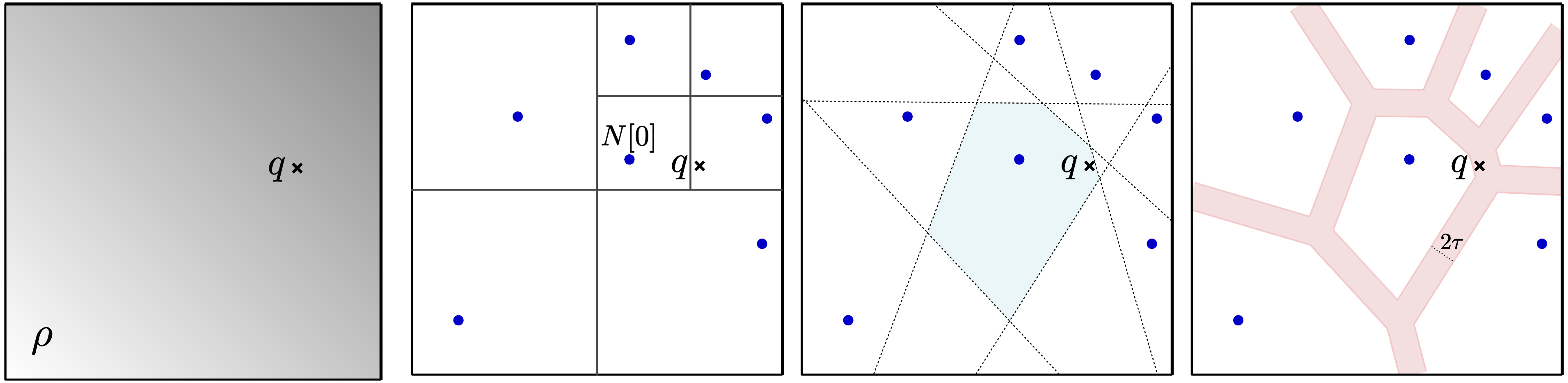
Procedural synthesis



F(x,y) called in every slice ‘pixel’

Procedural synthesis

F(x,y): is q=(x,y) inside?

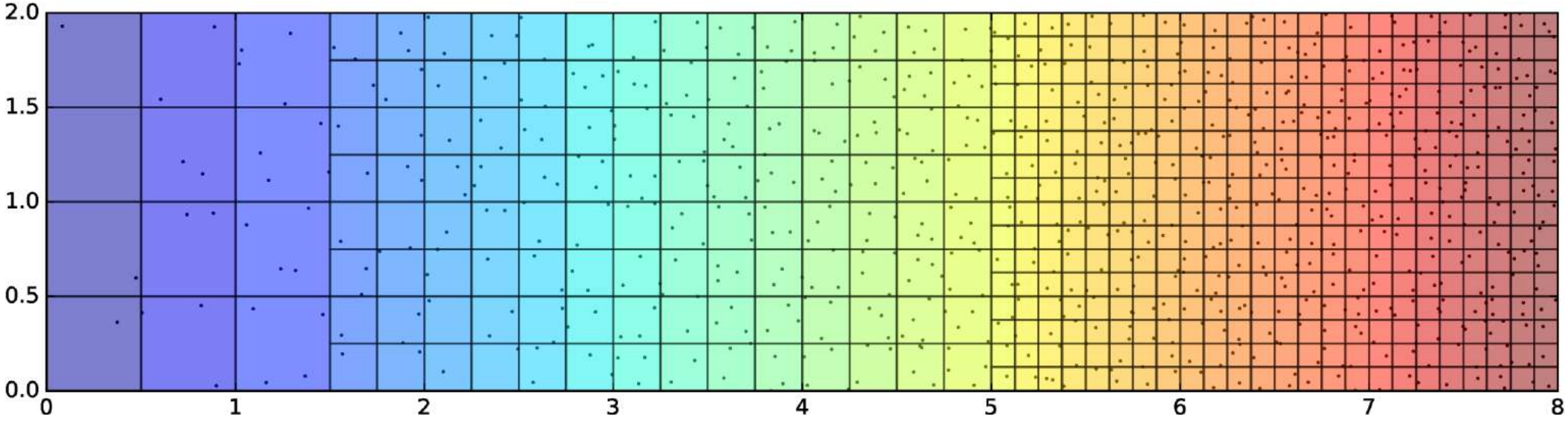


Target density | Neighboring seeds | Bisectors | Voronoi edges

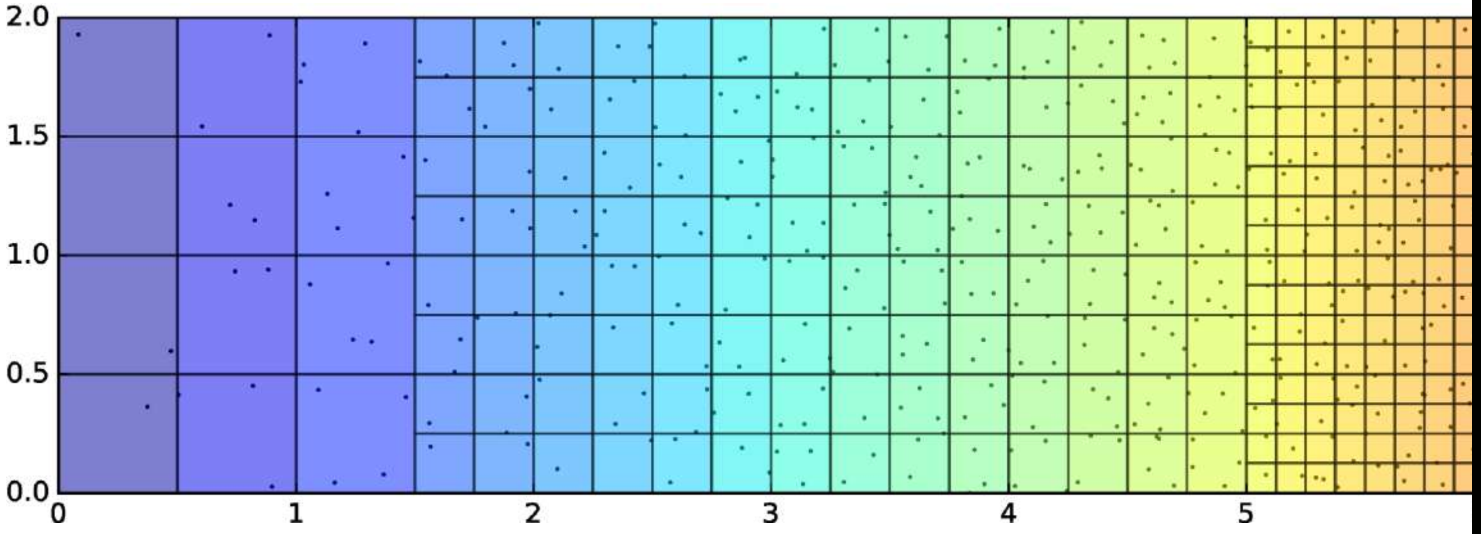
Local computations, O(1)

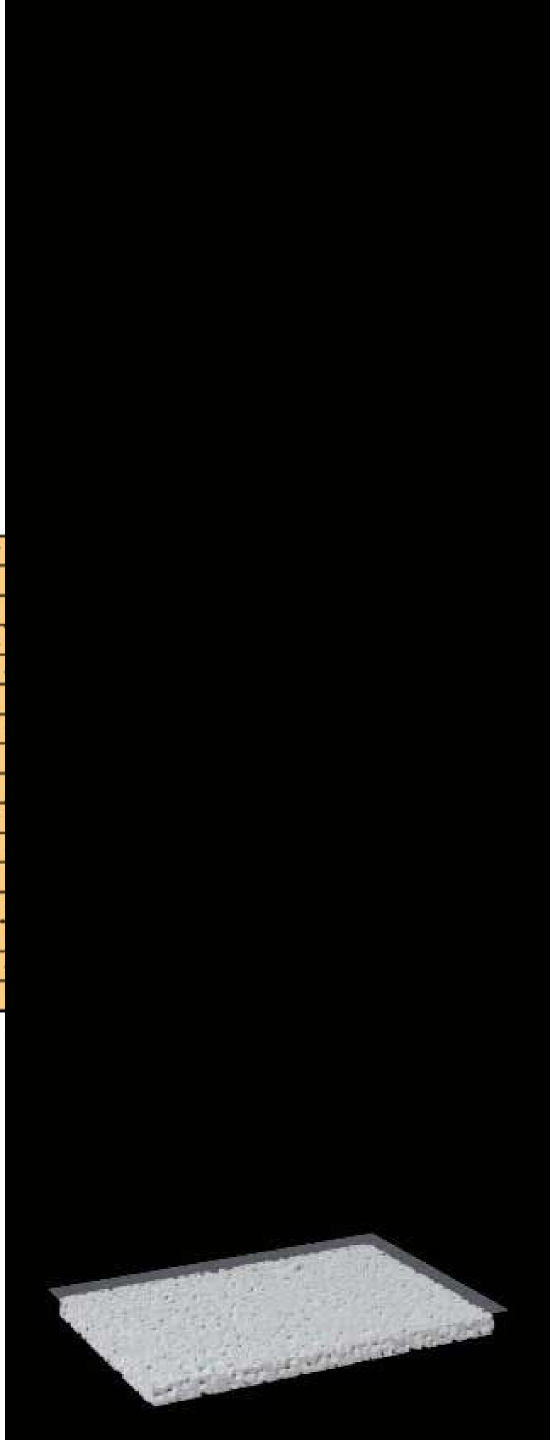
Trivialy parallel (GPU)

Gradation (stackless)

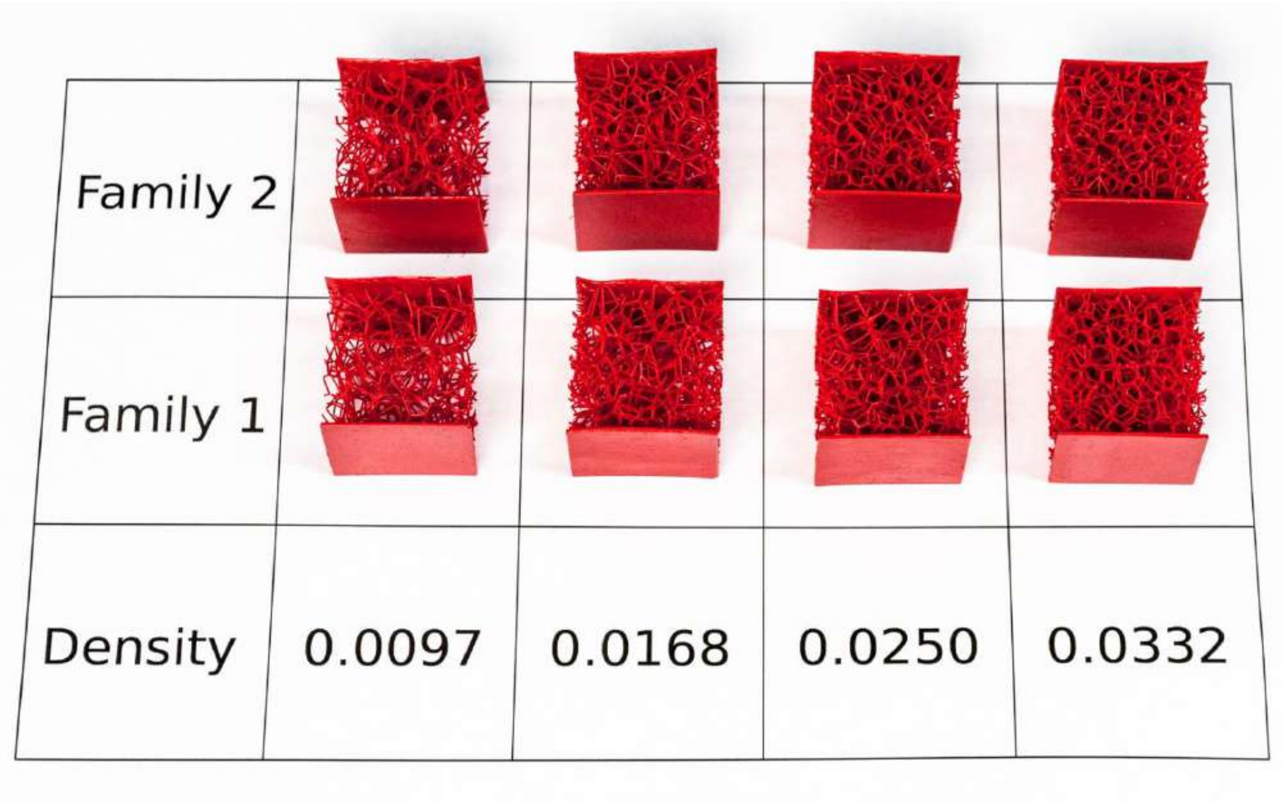


Gradation (stackless)

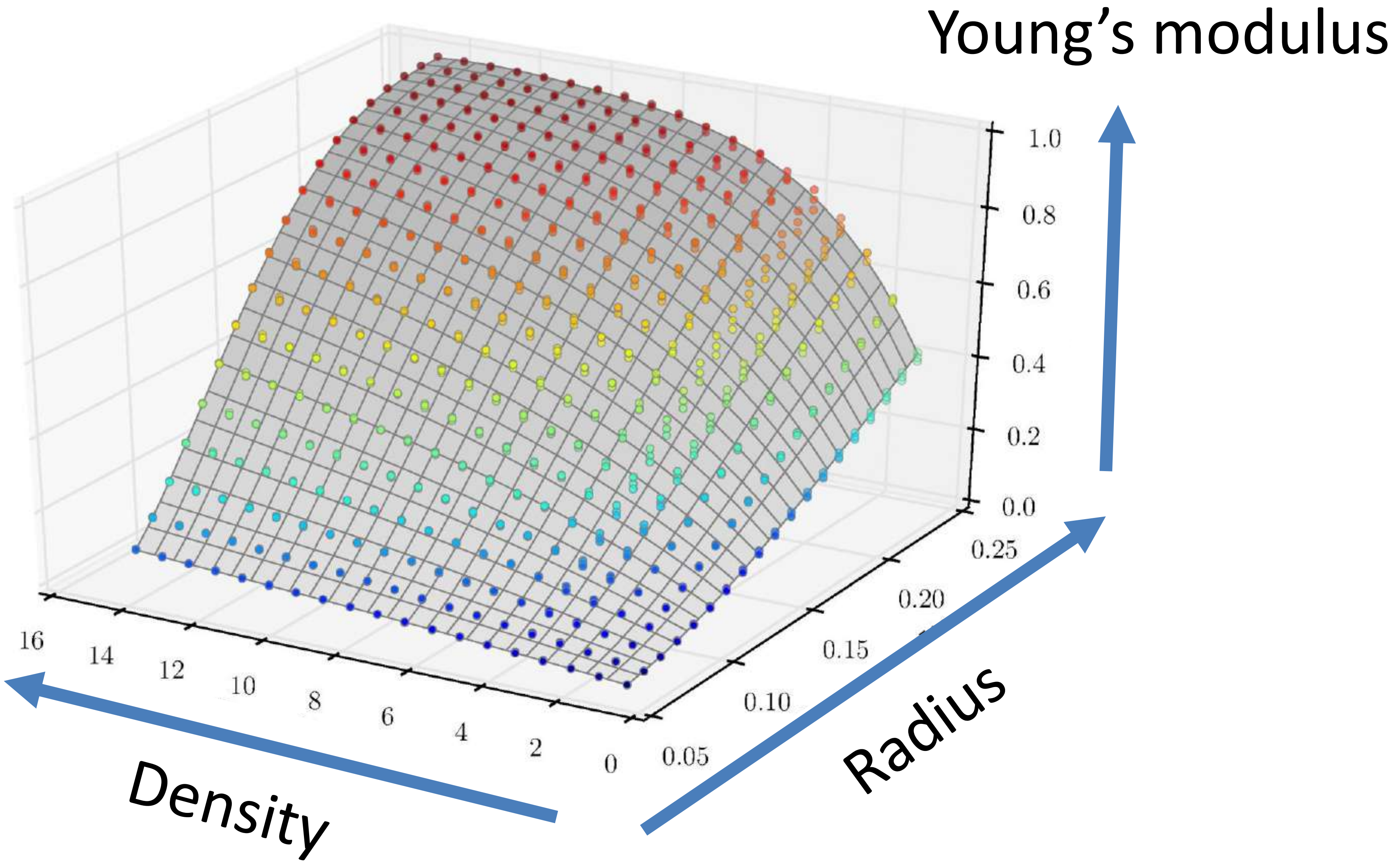




Elasticity control



Homogenisation



- **Results**

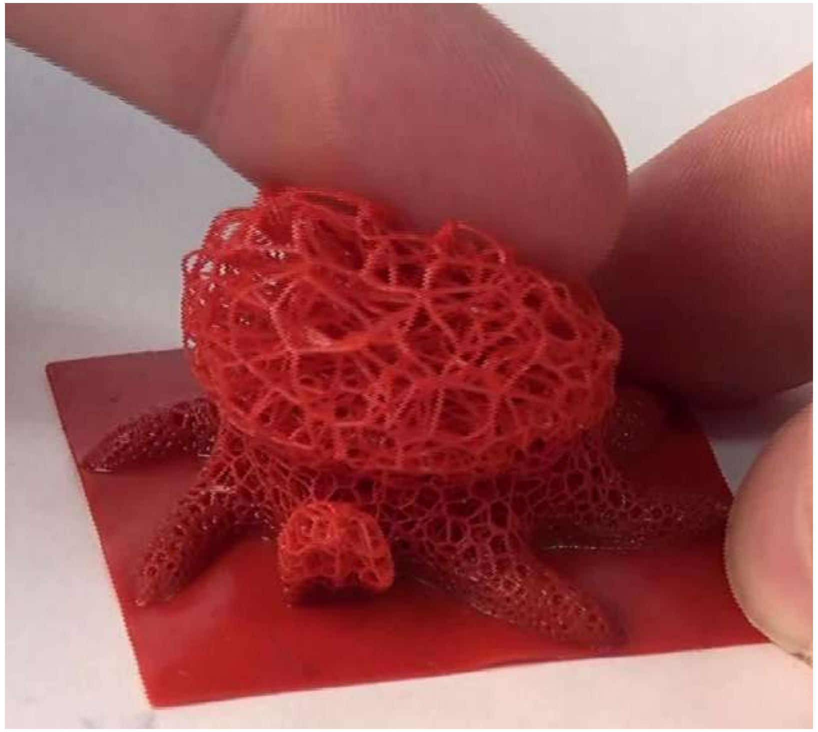
Crusty Knight

- **Results**

Articulated Finger

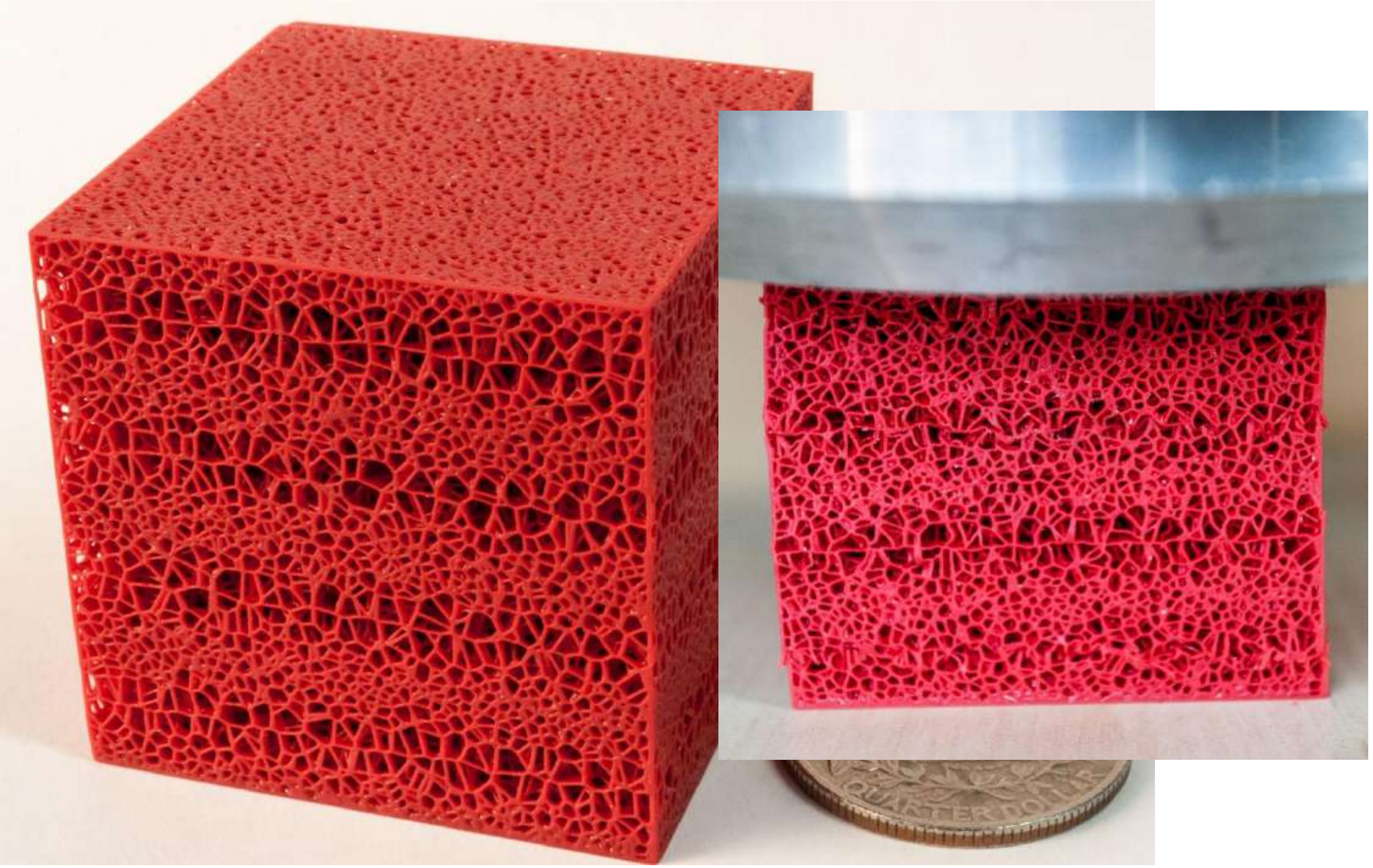
Cute Octopus

- Results



- **Results**

Anisotropy



Performances

Example		Extent (mm)	# Voxels	Volume	% Filtered	Time per slice (ms)
Moomin	fig. 1	$26.7 \times 40.8 \times 51.9$	$534 \times 815 \times 1038$	6.44%	0.005%	68.34
Ellipsoid	fig. 13	$30.9 \times 30.9 \times 41.1$	$617 \times 617 \times 822$	6.30%	0.001%	37.28
Knight	fig. 14	$26.1 \times 30.0 \times 50.55$	$521 \times 600 \times 1011$	12.50%	0.023%	20.25
Finger	fig. 15	$25.0 \times 23.25 \times 70.5$	$500 \times 465 \times 1410$	23.35%	0.006%	28.03
SIGGRAPH logo	fig. 16	$20.0 \times 40.0 \times 80.0$	$400 \times 800 \times 1600$	5.73%	0.003%	69.18
Half-dome	fig. 17	$25.0 \times 50.0 \times 25.0$	$500 \times 1000 \times 500$	19.49%	0.025%	71.22
Octopus	fig. 18	$41.7 \times 41.1 \times 28.8$	$833 \times 822 \times 576$	17.27%	0.009%	150.22
Anisotropic cube	fig. 19	$40.0 \times 40.0 \times 40.0$	$800 \times 800 \times 800$	26.86%	0.005%	113.52
Forest dragon	fig. 20	$770.1 \times 990.7 \times 961.7$	$15402 \times 19814 \times 19234$	N/A	N/A	1666.91

Thank you for your attention!

Questions?

Sylvain Lefebvre
sylvain.Lefebvre@inria.fr

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